Is there a flatness problem in classical cosmology?

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Overview

- Basic cosmology

- The qualitative flatness problem

- The quantitative flatness problem
  - collapsing models
  - nearly critical models
  - freely expanding models

- So what?
\[
\dot{R}^2 = \frac{8\pi G \rho R^2}{3} + \frac{\Lambda R^2}{3} - kc^2
\]

where \( k = -1, 0, +1 \) depending on curvature

\[
H := \frac{\dot{R}}{R}
\]

\[
\lambda := \frac{\Lambda}{3H^2}
\]

\[
\Omega := \frac{\rho}{\rho_{\text{crit}}} \equiv \frac{8\pi G \rho}{3H^2}
\]

\[
K := \Omega + \lambda - 1
\]

\[
q := \frac{-\dot{R}R}{\dot{R}^2} \equiv \frac{-\dot{R}}{RH^2} \equiv \frac{\Omega}{2} - \lambda
\]

\[
R = \frac{c}{H} \frac{\text{sign}(k)}{\sqrt{|\Omega + \lambda - 1|}} = \frac{c \text{ sign}(k)}{H} \frac{1}{\sqrt{|K|}}
\]
\[ \dot{R}^2 = \dot{R}_0^2 \left( \frac{\Omega_0 R_0}{R} + \frac{\lambda_0 R^2}{R_0^2} - K_0 \right) \]

\[ \left( \frac{\dot{R}}{R} \right)^2 = \left( \frac{\dot{R}_0}{R_0} \right)^2 \left( \frac{\Omega_0 R_0^3}{R^3} + \lambda_0 - \frac{K_0 R_0^2}{R^2} \right) \]

\[ H^2 = H_0^2 \left( \frac{\Omega_0 R_0^3}{R^3} + \lambda_0 - \frac{K_0 R_0^2}{R^2} \right) \]

\[ \lambda = \lambda_0 \left( \frac{H_0}{H} \right)^2 \]

\[ \rho = \left( \frac{\rho_0 R_0}{R} \right)^3 \]

\[ \Omega = \Omega_0 \left( \frac{H_0}{H} \right)^2 \left( \frac{R_0}{R} \right)^3 \]
possible worlds
\[ \alpha \]
random figure from the internet
Qualitative Flatness Problem

Is a ‘fine tuning’
of initial conditions required?

• Definition

  – Since $\Omega = 1$ is an unstable fixed point, why isn’t $\Omega$ very large or very small today?

  – Similar question for $\lambda$!

  – Generic ‘problem’ is not departure from $k = 0$ but rather departure from the Einstein-de Sitter model.

  – In general, always think ‘$\Omega$ and $\lambda$’ when you see $\Omega$, both later in this talk and in most of the literature.
• Easy (but most important) solution: For any value of $\Omega$ we observe, we can always find a time in the past when $\Omega$ was arbitrarily close to 1. Thus, the problem exists whatever value of $\Omega$ we observe today, or it doesn’t exist at all, but just reflects the boundary conditions of the Friedmann equations.

• Trivial solution: Einstein-de Sitter model must hold (now ruled out observationally but not long ago popular to the extent of dogma), however (e.g. Coles & Ellis):

  – We do live at a special time.

  – What is the probability distribution of initial conditions? (See papers by Coles and Evrard.)

  – $\Omega = 1$ is an *unstable* fixed point.
Quantitative Flatness Problem

Should we surprised that $\Omega_0 \approx 1$?

- Collapsing models
- Nearly critical models
- Freely expanding models
Collapsing Models

- Cosmological parameters evolve to infinity in a finite time $\implies$ flat distribution impossible.

- What range of parameter space is traversed during what fraction of the lifetime of the universe?

- Original argument?
age fraction

contours, right to left: 0.5, 0.6, 0.7, 0.8, 0.9
• ∃ constant of motion $\alpha = \text{sign}(K) \frac{27\Omega^2 \lambda}{4K^3}$.

• As far as I know, this was first used in the context of the flatness problem by Kayll Lake.

• This is a natural parameter to distinguish trajectories in the $\lambda$-$\Omega$ plane.
contours: 0.2, 0.4 ... 2, 3 ... 10, 20, 50, 100, 200, 500, 1000, 2000
contours: 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100
Quantitative Flatness Problem

Results

- Collapsing models
  - mathematically straightforward
  - independent of $H_0$, astrophysics
  - allowed range is ‘large’ but proves the point

- Nearly critical models
  - based on improbability of fine-tuning
  - independent of $H_0$, astrophysics
  - allowed range very small
• Freely expanding models
  – based on weak anthropic principle
  – assumptions about $H_0$, astrophysics
  – excludes very small values of $\Omega_0$ (as well as $\lambda_0$ very close to 1)

• If one a) assumes that $\lambda > 0$ and b) considers only the ‘weak flatness problem’ (should we wonder that the universe is nearly flat), then Lake’s fine-tuning argument is all one needs.
• G. Evrard & P. Coles: ‘Getting the Measure of the Flatness Problem’

• P. Coles & G. F. R. Ellis: *Is the Universe Open or Closed?*
  Cambridge: CUP (1997)

• K. Lake: ‘The Flatness Problem and Λ’

• P. Helbig: ‘Is there a flatness problem in classical cosmology?’