Is there a flatness problem in classical cosmology?

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Overview

- Basic cosmology
- The qualitative flatness problem
- The quantitative flatness problem
  - collapsing models
  - nearly critical models
  - freely expanding models
- So what?

Basic Cosmology

$$\dot{R}^2 = \frac{8\pi G\rho R^2}{3} + \frac{\Lambda R^2}{3} - kc^2$$

where $k = -1,\ 0,\ +1$ depending on curvature

$$H := \frac{\dot{R}}{R}$$

$$\lambda := \frac{\Lambda}{3H^2}$$

$$\Omega := \frac{\rho}{\rho_{\text{crit}}} \equiv \frac{8\pi G\rho}{3H^2}$$

$$K := \Omega + \lambda - 1$$

$$q := \frac{-\ddot{R}R}{\dot{R}^2} \equiv \frac{-\ddot{R}}{RH^2} \equiv \frac{\Omega}{2} - \lambda$$

$$R = \frac{c}{H}\frac{\text{sign}(k)}{\sqrt{|\Omega + \lambda - 1|}} = \frac{c}{H}\frac{\text{sign}(k)}{\sqrt{|K|}}$$

$$\dot{R}^2 = \dot{R}_0^2 \left(\frac{\Omega_0 R_0}{R} + \frac{\lambda_0 R^2}{R_0^2} - K_0 \right)$$

$$\left(\frac{\dot{R}}{R} \right)^2 = \left(\frac{\dot{R}_0}{R_0} \right)^2 \left(\frac{\Omega_0 R_0^3}{R^3} + \lambda_0 - \frac{K_0 R_0^2}{R^2} \right)$$

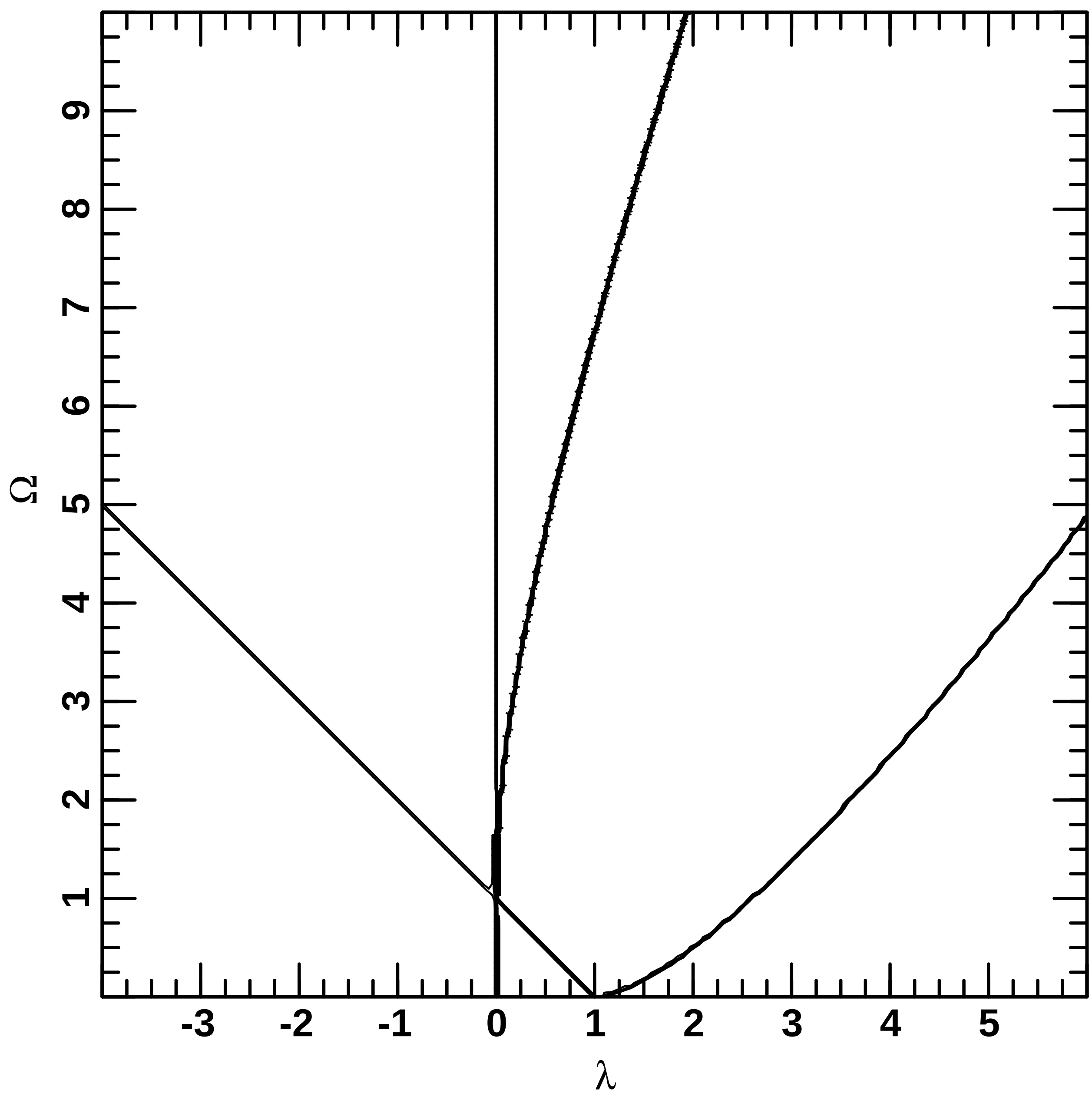
$$H^2 = H_0^2 \left(\frac{\Omega_0 R_0^3}{R^3} + \lambda_0 - \frac{K_0 R_0^2}{R^2} \right)$$

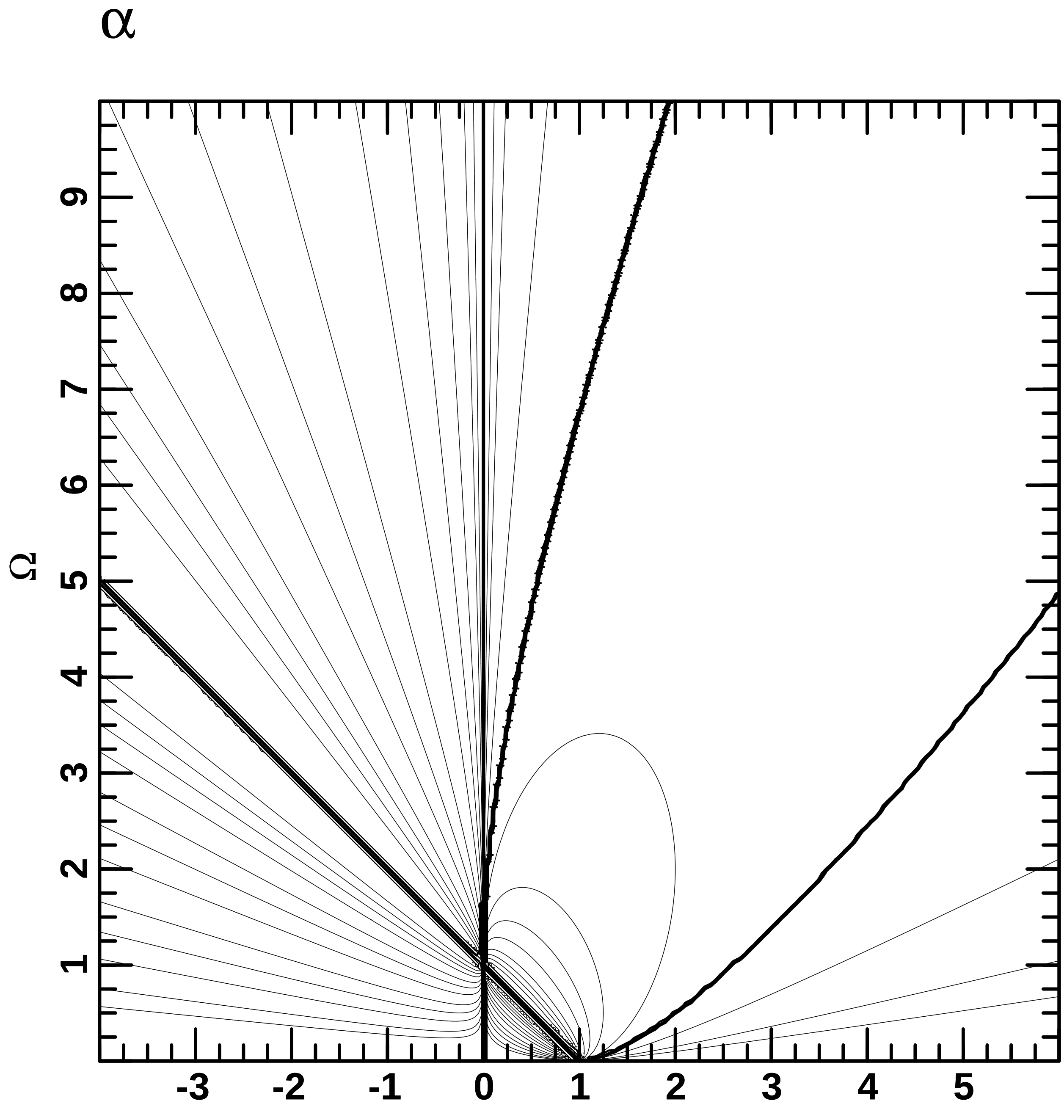
$$\lambda = \lambda_0 \left(\frac{H_0}{H} \right)^2$$

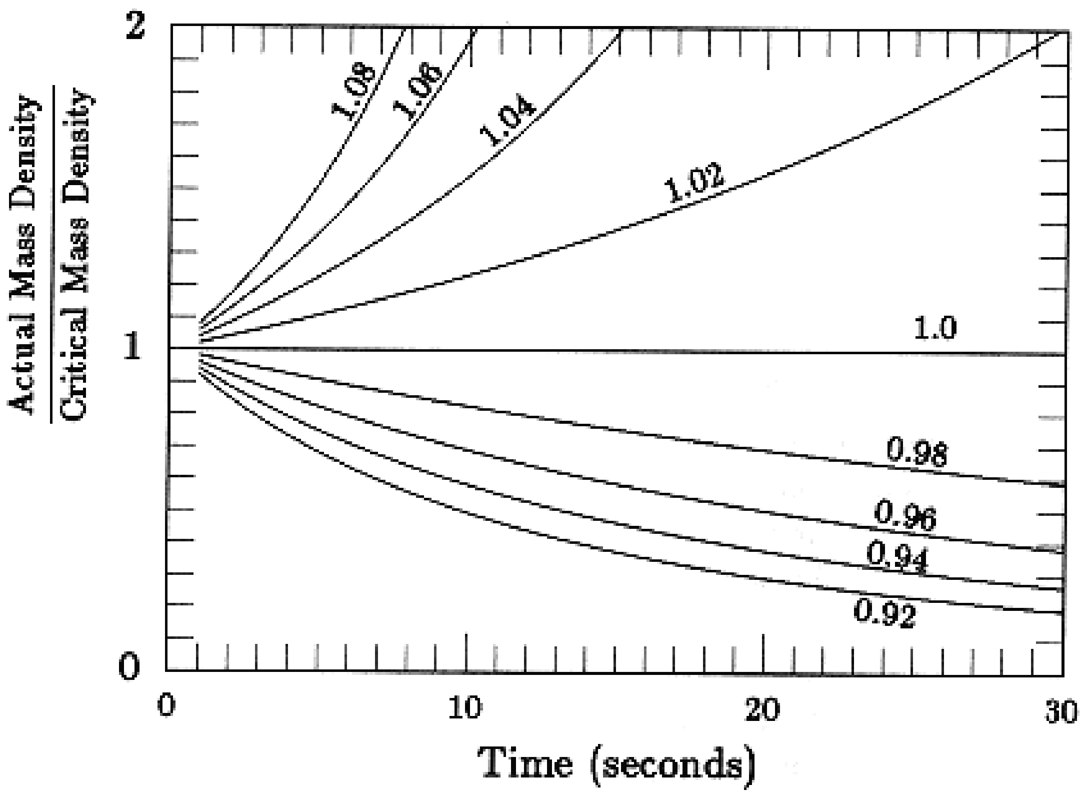
$$\rho = \left(\frac{\rho_0 R_0}{R} \right)^3$$

$$\Omega = \Omega_0 \left(\frac{H_0}{H} \right)^2 \left(\frac{R_0}{R} \right)^3$$

possible worlds




α
Ω
λ
-3
-2
-1
0
1
2
3
4
5
6
7
8
9



random figure from the internet

Qualitative Flatness Problem

Is a 'fine tuning'

of initial conditions required?

- Definition
  - Since $\Omega = 1$ is an unstable fixed point, why isn't Ω very large or very small today?
  - Similar question for λ!
  - Generic 'problem' is not departure from $k = 0$ but rather departure from the Einstein-de Sitter model.
  - In general, always think 'Ω and λ' when you see Ω, both later in this talk and in most of the literature.

- Easy (but most important) solution: For any value of Ω we observe, we can always find a time in the past when Ω was arbitrarily close to 1. Thus, the problem exists whatever value of Ω we observe today, or it doesn't exist at all, but just reflects the boundary conditions of the Friedmann equations.

- Trivial solution: Einstein-de Sitter model must hold (now ruled out observationally but not long ago popular to the extent of dogma), however (e.g. Coles & Ellis):

  – We do live at a special time.

  – What is the probabibility distribution of initial conditions? (See papers by Coles and Evrard.)

  – $\Omega = 1$ is an *unstable* fixed point.

Quantitative Flatness Problem

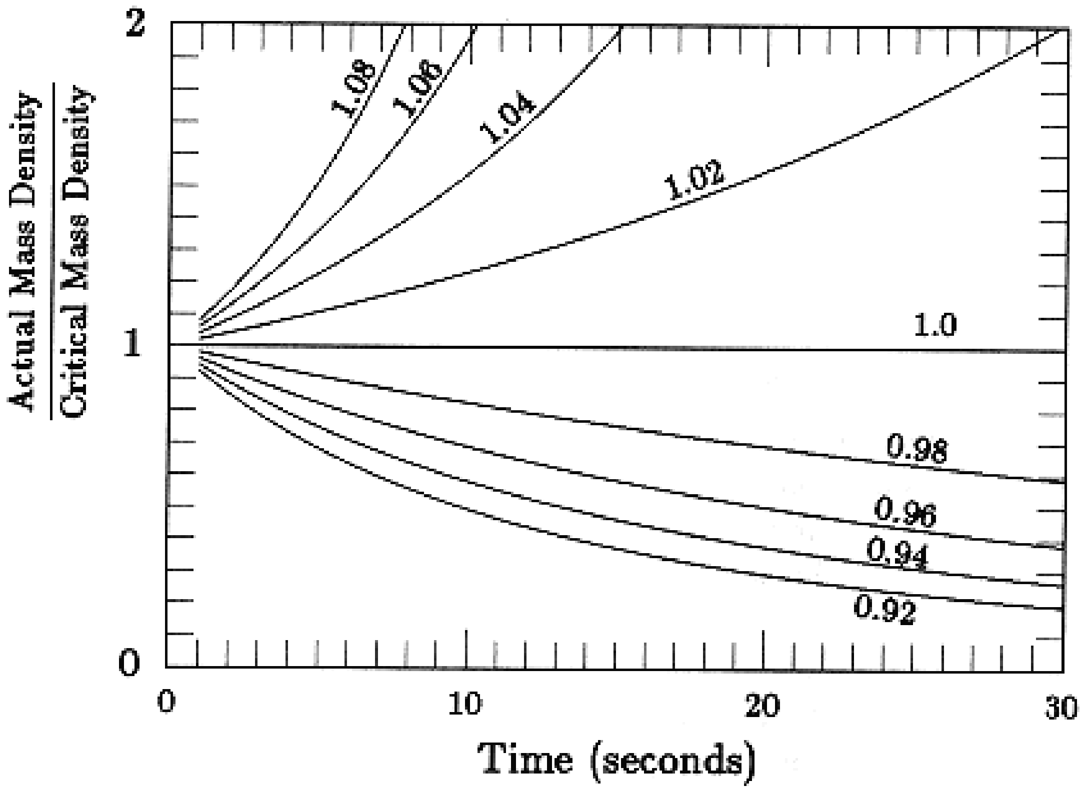
Should we surprised

that $\Omega_0 \approx 1$?

- Collapsing models

- Nearly critical models

- Freely expanding models



2
1
0
Actual Mass Density
Critical Mass Density
1.08
1.06
1.04
1.02
1.0
0.98
0.96
0.94
0.92
0
10
20
30
Time (seconds)

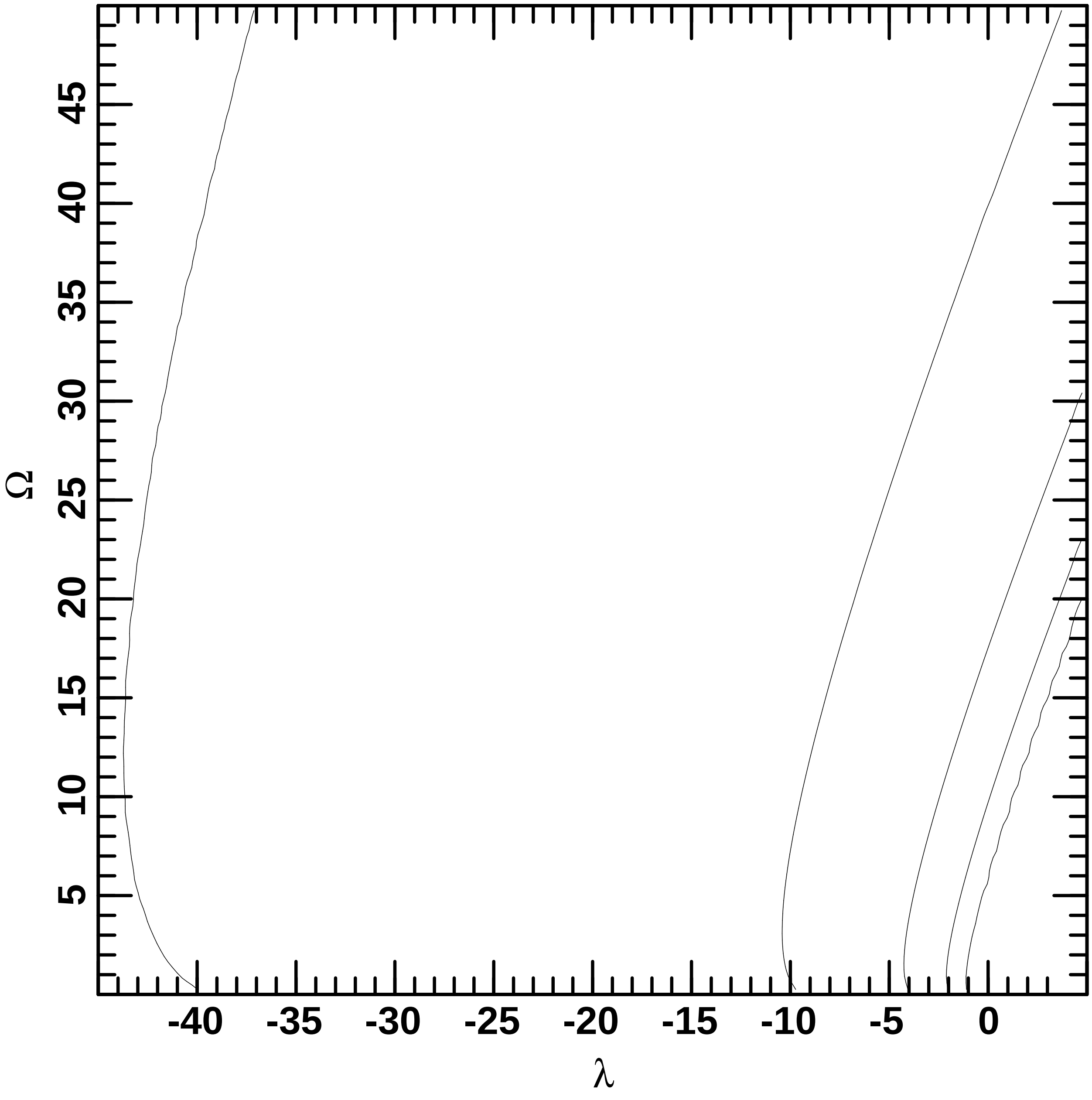
Collapsing Models

- Cosmological parameters evolve to infinity in a finite time $\Longrightarrow$ flat distribution impossible.

- What range of parameter space is traversed during what fraction of the lifetime of the universe?

- Original argument?

age fraction



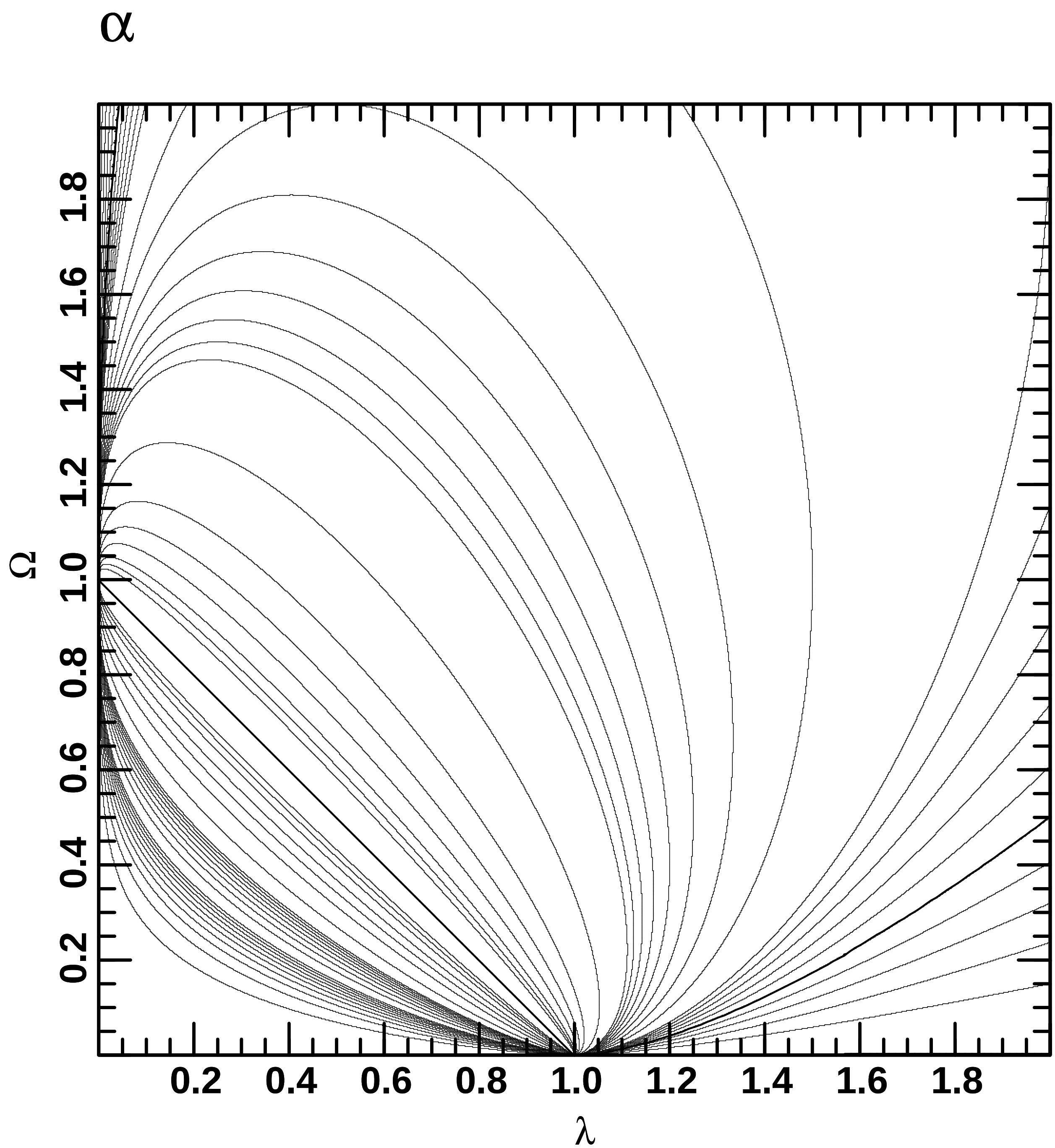
contours, right to left: 0.5, 0.6, 0.7, 0.8, 0.9

Nearly Critical Models

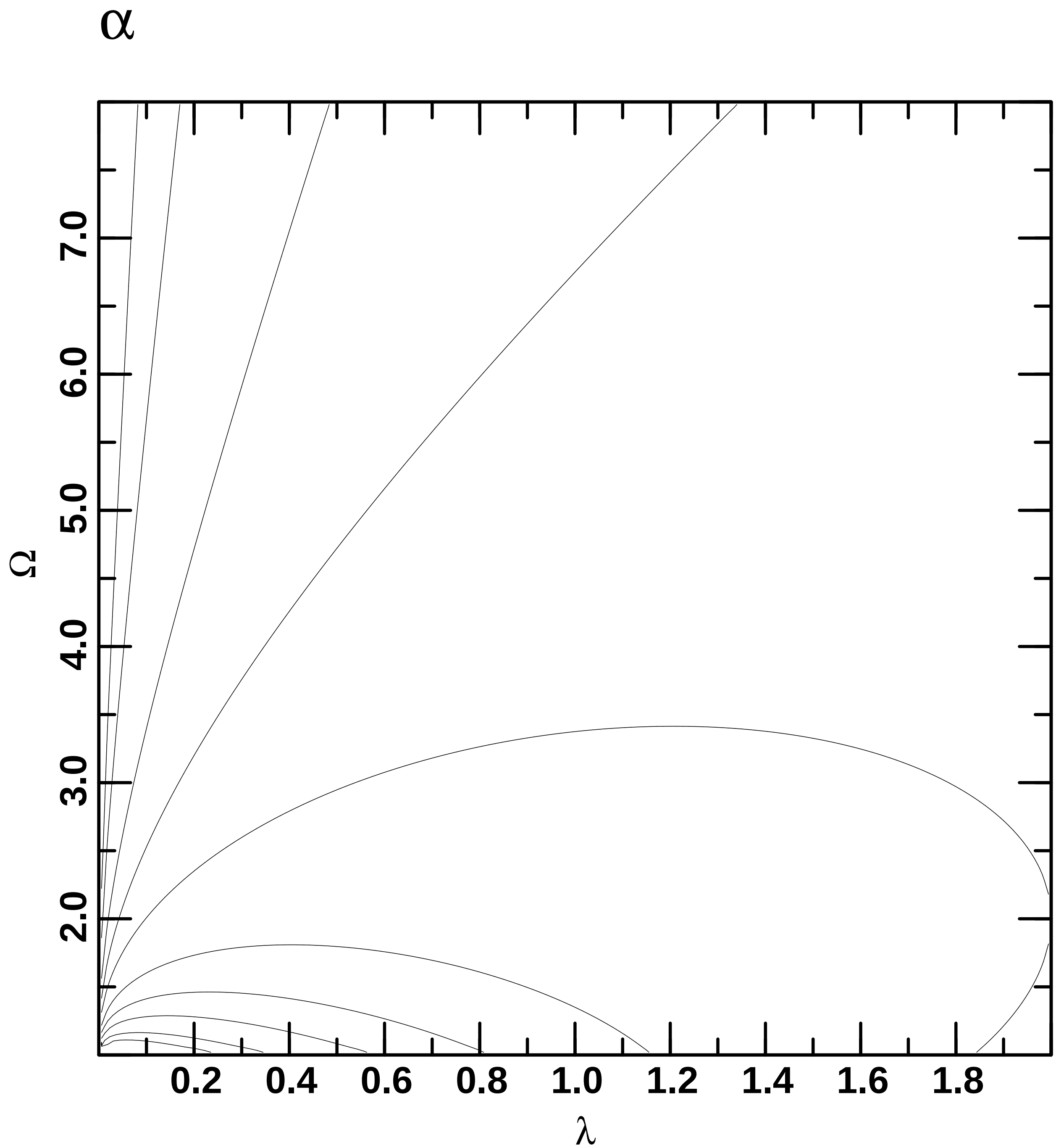
- $\exists$ constant of motion $\alpha = \text{sign}(K)\frac{27\Omega^2\lambda}{4K^3}$.

- As far as I know, this was first used in the context of the flatness problem by Kayll Lake.

- This is a natural parameter to distinguish trajectories in the λ-Ω plane.



contours: 0.2, 0.4 ... 2, 3 ... 10, 20, 50, 100, 200, 500, 1000, 2000



contours: 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100

Quantitative Flatness Problem

Results

- Collapsing models
  - mathematically straightforward
  - independent of H_0, astrophysics
  - allowed range is ‘large’ but proves the point
- Nearly critical models
  - based on improbability of fine-tuning
  - independent of H_0, astrophysics
  - allowed range very small

- Freely expanding models
  - based on weak anthropic principle
  - assumptions about H_0, astrophysics
  - excludes very small values of Ω_0 (as well as λ_0 very close to 1)

- If one a) assumes that $\lambda > 0$ and b) considers only the 'weak flatness problem' (should we wonder that the universe is nearly flat), then Lake's fine-tuning argument is all one needs.

More details

- G. Evrard & P. Coles: ‘Getting the Measure of the Flatness Problem’
  CQG, **12**, 10, L93-98 (1995)

- P. Coles & G. F. R. Ellis: *Is the Universe Open or Closed?*
  Cambridge: CUP (1997)

- K. Lake: ‘The Flatness Problem and Λ’
  PRL, **94**, 20, 201102 (2005)

- P. Helbig: ‘Is there a flatness problem in classical cosmology?’
  MNRAS, **421**, 1, 561–569 (2012)