Black Hole Collisions in Asymptotically de Sitter Spacetimes

Miguel Zilhão, Vitor Cardoso, Leonardo Gualtieri, Carlos Herdeiro,
Ulrich Sperhake, and Helvi Witek

Abstract We report on the first dynamical evolutions of black holes in asymptotically de Sitter spacetimes. We focus on the head-on collision of equal mass binaries and compare analytical and perturbative methods with full blown nonlinear simulations. Our results include an accurate determination of the merger/scatter transition (consequence of an expanding background) for small mass binaries and a test of the Cosmic Censorship conjecture, for large mass binaries. We observe that, even starting from small separations, black holes in large mass binaries eventually lose causal contact, in agreement with the conjecture.

Miguel Zilhão
Centro de Física do Porto, Departamento de Física e Astronomia
Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre, 4169-007 Porto, Portugal
Center for Computational Relativity and Gravitation, Rochester Institute of Technology,
Rochester, NY 14623, e-mail: mzilhao@fc.up.pt

Miguel Zilhão and Carlos Herdeiro
Departamento de Física da Universidade de Aveiro
Campus de Santiago, 3810-183 Aveiro, Portugal

Vitor Cardoso
Department of Physics and Astronomy, The University of Mississippi
University, MS 38677-1848, USA

Leonardo Gualtieri
Dipartimento di Fisica, "Sapienza" Università di Roma & Sezione INFN Roma1
Piazzale Aldo Moro 5, 00185, Roma, Italy

Ulrich Sperhake
Institut de Ciències de l'Espai (CSIC-IEEC), Facultat de Ciències
Campus UAB, E-08193 Bellaterra, Spain
California Institute of Technology, Pasadena, CA 91125, USA;

Vitor Cardoso, Ulrich Sperhake and Helvi Witek
Centro Multidisciplinar de Astrofísica — CENTRA, Departamento de Física
Instituto Superior Técnico — IST, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

Ulrich Sperhake and Helvi Witek
DAMTP, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, UK

1 Introduction

Nonlinear dynamics in cosmological backgrounds has the potential to teach us immensely about our universe, and also to serve as prototype for nonlinear processes in generic curved spacetimes. *de Sitter* spacetime is the simplest accelerating universe—a maximally symmetric solution of Einstein's equations with a positive cosmological constant—which seems to model quite well the present cosmological acceleration [1].

Key questions concerning the evolution towards a de Sitter, spatially homogeneous universe are how inhomogeneities develop in time and, in particular, if they are washed away by the cosmological expansion [2]. Answering them requires controlling the imprint of the gravitational interaction between localised objects on the large-scale expansion. Conversely, the cosmological dynamics should leave imprints in strong gravitational phenomena like primordial black hole formation [3] or the gravitational radiation emitted in a black hole binary coalescence, which carry signatures of the cosmological acceleration as it travels across the universe. Identifying these signatures is not only of conceptual interest but also phenomenologically relevant, in view of the ongoing efforts to directly detect gravitational radiation.

Finally, dynamics in asymptotically de Sitter spacetimes could also teach us about more fundamental questions such as cosmic censorship: two black holes of sufficiently large mass in de Sitter spacetime would, upon merger, give rise to too large a black hole to fit in its cosmological horizon. In this case the end state would be a naked singularity. This possibility begs for a time evolution of such a configuration. Does the time evolution of non-singular data containing two black holes result in a naked singularity, or are potentially offending black holes simply driven away from each other by the cosmological expansion?

Following [4], we here report on numerical evolutions of black hole binaries in an asymptotically de Sitter geometry. Even though we consider a range of values for the cosmological constant far larger than those which are phenomenologically viable, these results provide useful insight on the general features of dynamical black hole processes in spacetimes with a cosmological constant, which can improve our understanding of our universe.

1.1 Schwarzschild-de Sitter

The Schwarzschild-de Sitter spacetime, written in static coordinates reads

$$\mathrm{d}s^2 = -f(R)\mathrm{d}T^2 + f(R)^{-1}\mathrm{d}R^2 + R^2\mathrm{d}\Omega_2\,. \tag{1}$$

The solution is characterised by two parameters: the black hole mass m and the Hubble parameter H,

$$f(R) = 1 - 2m/R - H^2R^2\,, \qquad H \equiv \sqrt{\Lambda/3}\,. \tag{2}$$

$f(R)$ has two zeros, at $R = R_\pm$, $R_- < R_+$, if

$$0 < mH < mH_{\rm crit}\ , \qquad mH_{\rm crit} \equiv \sqrt{1/27}\ . \tag{3}$$

These zeros are the location of the black hole event horizon (R_-) and of a cosmological horizon (R_+). If $H = 0$, then $R_- = 2m$; if $m = 0$, then $R_+ = 1/H$. If $H, m \neq 0$, then $R_- > 2m$ and $R_+ < 1/H$.

The basic dynamics in this spacetime may be inferred by looking at radial timelike geodesics. They obey the equation $(\mathrm{d}R/\mathrm{d}\tau)^2 = E^2 - f(R)$, where τ is the proper time and E is the conserved quantity associated to the Killing vector field $\partial/\partial T$. In the static patch ($R_- < R < R_+$), E can be regarded as energy. From this equation we see that $f(R)$ is an effective potential. This potential has a maximum at

$$R_{\rm max} = (m/H^2)^{1/3}\,. \tag{4}$$

Geodesics starting from rest (i.e. $\mathrm{d}R/\mathrm{d}\tau(\tau = \tau_0) = 0$) will fall into the black hole if $R_- < R < R_{\rm max}$ or move away from the black hole if $R_{\rm max} < R < R_+$.

As we will discuss in the next section, the initial data for an evolution in the de Sitter universe can be computed in a similar manner as has been done in asymptotically flat space as long as one chooses a foliation with extrinsic curvature K_{ij} having only a trace part [5, 6]. Such a coordinate system is known for Schwarzschild-de Sitter: *McVittie coordinates* [7]. These are obtained from static coordinates by the transformation $(T, R) \to (t, r)$ given by

$$R = (1+\xi)^2 a(t) r\ , \qquad T = t + H \int \frac{R\,\mathrm{d}R}{f(R)\sqrt{1-2m/R}}\ , \tag{5}$$

where $a(t) = \exp(Ht)$ and $\xi \equiv \frac{m}{2a(t)r}$. One obtains McVittie's form for Schwarzschild-de Sitter:

$$\mathrm{d}s^2 = -\left(\frac{1-\xi}{1+\xi}\right)^2 \mathrm{d}t^2 + a(t)^2 (1+\xi)^4 (\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2)\ . \tag{6}$$

By setting $m = 0$ in McVittie coordinates one recovers an FRW cosmological model with flat spatial curvature and an exponentially growing scale factor. The cosmological horizon $\mathscr{H}_C$ discussed above, located at $R = 1/H$, stands at $r_{\mathscr{H}_C} = 1/(He^{Ht})$. The spatial sections of $\mathscr{H}_C$ seem to be shrinking down in this coordinate system. What happens, in fact, is that the exponentially fast expansion is taking any observer to the outside of $\mathscr{H}_C$. This is a well known phenomenon in studies of inflation and has important consequences for the numerical evolution.

1.2 Numerical Setup

For the numerical implementation we make use of the Baumgarte, Shapiro, Shibata and Nakamura (BSSN) formulation, *e.g.* [8, 9] with the *moving puncture* technique [10, 11]. In terms of the BSSN variables $\chi, \tilde{\gamma}_{ij}, \tilde{A}_{ij}, \tilde{\Gamma}^k$ (cf. App. A in [9]), the evolution equations are the standard ones except for the one of the trace of the extrinsic curvature which becomes $\left(\partial_t - \mathscr{L}_\beta\right) K = [\dots] - \alpha\Lambda$, where $[\dots]$ denotes the standard right-hand side of the BSSN equations in the absence of source terms (see *e.g.* [8]). Moreover, a new variable $\bar{\chi} = \exp(2Ht)\chi$ was evolved instead of χ [2].

In references [5, 6] it was observed that imposing a spacetime slicing obeying $K^i{}_j = -H\delta^i_j$, and a spatial metric of the form $\mathrm{d}l^2 = \psi^4 \tilde{\gamma}_{ij} \mathrm{d}x^i \mathrm{d}x^j$, the equations to be solved in order to obtain initial data are equivalent to those in vacuum. In particular, for a system of N black holes momentarily at rest (with respect to the given spatial coordinate patch), the conformal factor ψ takes the form

$$\psi = 1 + \sum_{i=1}^{N} \frac{m_i}{2|r - r_{(i)}|} \,. \tag{7}$$

There are $N+1$ asymptotically de Sitter regions, as $|r - r_{(i)}| \to 0, +\infty$; the total mass for observers in the common asymptotic region ($|r - r_{(i)}| \to +\infty$) is $\sum_i m_i$ [6].

1.3 Numerical Results

For binary black hole initial data, we start by reproducing the results of Nakao et. al [6], where the critical distance between two black holes for the existence of a common Black hole Apparent Horizon (BAH) already at $t = 0$ was studied. We thus prepare initial data (7) with $m_1 = m_2$ and take all quantities in units of the total mass $m = m_1 + m_2$. The two punctures are set initially at symmetric positions along the z axis. The critical value for the cosmological constant, for which the black hole and cosmological horizon coincide is now $mH_{\rm crit} = 1/\sqrt{27}$. We call *small (large) mass binaries* those, for which $H < H_{\rm crit}$ ($H > H_{\rm crit}$). Our results for the critical separation in small mass binaries, at $t = 0$, as function of the Hubble parameter are shown in Fig. 1. The line (diamond symbols) agrees, after a necessary normalisation, with Fig. 14 of [6].

We now consider head-on collisions of two black holes with no initial momentum, i.e. the time evolution of these data. For subcritical Hubble constant $H < H_{\rm crit} = 1/(\sqrt{27}m)$, we monitor the evolution of the areal radius of the BAHs and that of the Cosmological Apparent Horizons (CAHs) of an observer at $z = 0$. For instance, for $H = 0.9H_{\rm crit}$ and proper (initial) separation $3.69m$ we find that the areal radii of the BAH and CAH are approximately constant and equal to $R_{\rm BAH} \simeq 2.36m$ and $R_{\rm CAH} \simeq 4.16m$, respectively. As expected the two initial BAHs, as well as the final horizon, are inside the CAH. As a comparison, a Schwarzschild-de Sitter spacetime with the same H has $R_{\rm BAH} \simeq 2.43m$ and $R_{\rm CAH} \simeq 4.16m$. This suggests that the

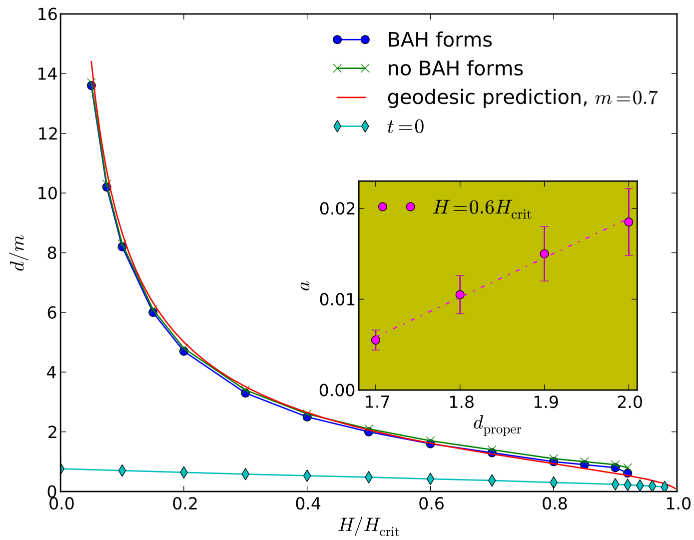

Fig. 1 Critical coordinate distance for small mass binaries as a function of $H/H_{\rm crit}$. We obtain this estimate from the coordinate distance to the horizon, Eq. (4), for a particular value of m. The $t = 0$ line refers to the critical separation between having or not having a common BAH in the initial data. The inset shows details of the approach to the critical line for $H = 0.6H_{\rm crit}$, where a is an acceleration parameter.

interaction effects (binding energy and emission of gravitational radiation) are of the order of a few per cent for this configuration.

By performing a large set of simulations for various cosmological parameters H and initial distance d, we have bracketed the critical distance for the merger/scatter region as a function of the Hubble parameter H for the "dynamical" case, i.e., the initial *coordinate* distance between the black holes such that no common BAH forms. The results are displayed in Fig. 1 (circles and $\times$ symbols).

As expected the critical distance becomes larger as compared to the initial data value ("$t = 0$" line): there are configurations for which a common BAH is absent in the initial data but appears during the evolution (just as in asymptotically flat spacetime). The numerical results can be qualitatively well approximated by a point particle prediction—from Eq. (4). To do such comparison a transformation to McVittie coordinates needs to be done; we have performed such transformation at McVittie time $t = 0$. Intriguingly, for a particular value of $m \simeq 0.7$, the point particle approximation matches quantitatively very well the numerical result; the curve obtained from the geodesic prediction in Fig. 1 is barely distinguishable from the numerical results.

A further interesting feature concerns the approach to the critical line. For an initially static binary close to the critical initial separation, the coordinate distance

d scales as $d = d_0 + at^2$. In general the acceleration parameter scales as $\log a = C + \Gamma \log(d - d_0)$, where $\Gamma = 1$ in the geodesic approximation. A fit to our numerical results for $H = 0.6H_{\rm crit}$ (dashed curve in the inset of Fig. 1) for example yields $C = -3.1$, $\Gamma = 0.9$ in rough agreement with this expectation. Details of this regime are given in the inset of Fig. 1.

Finally, we have performed evolutions with $H > H_{\rm crit}$. On the assumption of weak gravitational wave release, such evolutions can test the cosmic censorship conjecture since the observation of a merger in such case would reveal a violation of the conjecture [13]. From general arguments and from the simulations with $H < H_{\rm crit}$, we know the cosmological repulsion will dominate for sufficiently large initial distance and in that case we can even expect that a CAH for the observer at $z = 0$ will not encompass the BAHs. This indicates the black holes are no longer in causal contact and therefore can never merge. Our numerical results confirm this overall picture. To test the potentially dangerous configurations, we focus on the regime in which the black holes are initially very close. A typical example is depicted in Fig. 2, for a supercritical cosmological constant $H = 1.05H_{\rm crit}$, and an initial coordinate distance $d/m = 1.5002$. Even though the initial separation is very small, we find that the holes move *away* from each other, with a proper separation increasing as the simulation progresses. In fact, further into the evolution, a distorted CAH appears, and remains for as long as the simulation lasts. The evolution therefore indicates that the spacetime becomes, to an excellent approximation, empty de Sitter space for the observer at $z = 0$ and that the black holes are not in causal contact. Observe

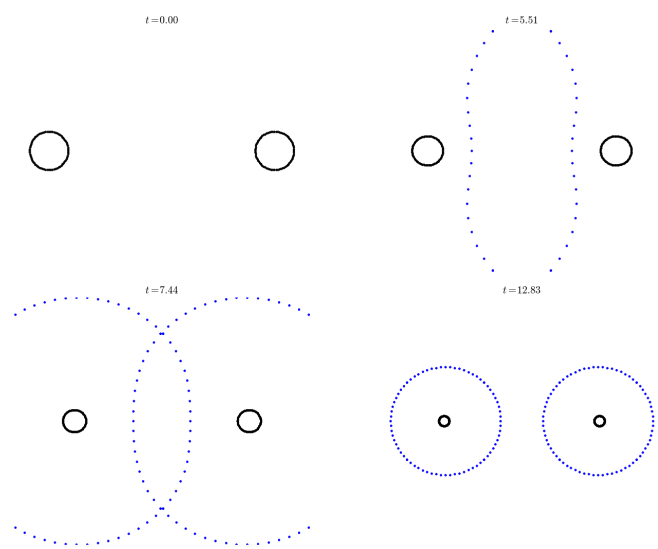

Fig. 2 Snapshots at different times of a simulation with $H = 1.05H_{\rm crit}$, and an initial coordinate distance $d/m = 1.5002$. The dotted blue line denotes the CAHs (for the observers moving with the black holes). The full animation for this simulation can be found in [12].

that qualitatively similar evolutions can be found in small mass binaries when the initial distance is larger than the critical value

1.4 Final Remarks

We have presented evidence that the numerical evolution of black hole spacetimes in de Sitter universes is under control. Our results open the door to new studies of strong field gravity in cosmologically interesting scenarios. In closing, we would like to mention that our results are compatible with cosmic censorship in cosmological backgrounds. However, an analytic solution with multiple (charged and extremal) black holes in asymptotically de Sitter spacetime is known, and has been used to study cosmic censorship violations [14]. In *collapsing* universes a potential violation of the conjecture has been reported, although the conclusion relied on singular initial data. To clarify this issue, it would be of great interest to perform numerical evolution of large mass black hole binaries, analogous to those performed herein, but in collapsing universes. This will require adaptations of our setup, since the "expanding" behaviour discussed of the coordinate system will turn into a "collapsing" one, which raises new numerical challenges.

References

1. E. Komatsu, K.M. Smith, J. Dunkley, et al., *Seven-year Wilkinson microwave anisotropy probe (WMAP) Observations: Cosmological Interpretation*, Astrophys. J. Suppl. Ser. **192**, 18 (2011)
2. M. Shibata, K.i. Nakao, T. Nakamura, K.i. Maeda, *Dynamical evolution of gravitational waves in the asymptotically de Sitter space-time*, Phys. Rev. D **50**, 708 (1994)
3. M. Shibata, M. Sasaki, *Black hole formation in the Friedmann universe: Formulation and computation in numerical relativity*, Phys. Rev. D **60**, 084002 (1999)
4. M. Zilhão, V. Cardoso, L. Gualtieri, et al., *Dynamics of black holes in de Sitter spacetimes*, Phys. Rev. D **85**, 104039 (2012)
5. K.I. Nakao, T. Nakamura, K.I. Oohara, K.I. Maeda, *Numerical study of cosmic no-hair conjecture: Formalism and linear analysis*, Phys. Rev. D **43**, 1788 (1991)
6. K.i. Nakao, K. Yamamoto, K.i. Maeda, *Apparent horizons of an N-black-hole system in a space-time with a cosmological constant*, Phys. Rev. D **47**, 3203 (1993)
7. G.C. McVittie, *The mass-particle in an expanding universe*, Mon. Not. R. Astron. Soc. **93**, 325 (1933)
8. M. Alcubierre, *Introduction to 3+1 numerical relativity*, *International Series of Monographs on Physics*, vol. 140 (Oxford University Press, Oxford; New York, 2008)
9. U. Sperhake, *Binary black-hole evolutions of excision and puncture data*, Phys. Rev. D **76**, 104015 (2007)
10. M. Campanelli, C.O. Lousto, P. Marronetti, Y. Zlochower, *Accurate evolutions of orbiting black-hole binaries without excision*, Phys. Rev. Lett. **96**, 111101 (2006)
11. J.G. Baker, J. Centrella, D.I. Choi, M. Koppitz, J. van Meter, *Gravitational wave extraction from an inspiraling configuration of merging black holes*, Phys. Rev. Lett. **96**, 111102 (2006)
12. Black Hole Dynamics Beyond General Relativity. URL http://blackholes.ist.utl.pt/?page=Files#h-4

13. S.A. Hayward, T. Shiromizu, K.I. Nakao, *A cosmological constant limits the size of black holes*, Phys. Rev. D **49**, 5080 (1994)
14. D.R. Brill, G.T. Horowitz, D. Kastor, J.H. Traschen, *Testing cosmic censorship with black hole collisions*, Phys. Rev. D **49**, 840 (1994)