

The inflationary origin of the seeds of cosmic structure: quantum theory and the need for novel physics

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Abstract The Inflationary account for the emerging of the seeds of cosmic structure from quantum fluctuations is a central part of our current views of cosmology. It is, on the one hand, extremely successful at the phenomenological level, and yet it retains an aspect that is generally regarded as controversial: The exact mechanism by which quantum fluctuations transmute into actual inhomogeneities. We will review the considerations that lead us to conclude that the fully satisfactory resolution of the issue requires novel physics and we will discuss an option we have been considering in this regard.

1 Introduction

This conference commemorates the time spent by Einstein in Prague, which was instrumental in his development of General Relativity, a subject which has been the focus of the majority of the other presentations. I will be touching on the other great question that preoccupied Einstein at the time: Quantum Theory. We note that, the subject of this manuscript; inflation, represents the only generally accepted example of an instance in which General Relativity, Quantum Theory and observations come together. It is, therefore, quite remarkable that it is precisely here where we must confront the conceptual difficulties of quantum theory itself. In fact the ideas I will be exploring are strongly motivated by the arguments that R. Penrose [1] and L. Diósi [2] have been advancing regarding the collapse of the wave function as a dynamical process to be incorporated in a modified Schrödinger's equation, and the role that gravity might play in this.

In the present manuscript, and in contrast to other works [3, 4] where I have focused on the difficulties or shortcomings of the postures advocated in standard

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treatments of the emergence of the seeds of cosmic structure, I will be focusing on aspects that would be encountered independently of the conceptual approach one takes, as long as one attempts to provide a specific characterization of the various stages in the cosmological evolution. Moreover, I will show the connection, sometimes not easily recognized, between the actual treatment we have been using, and other approaches to deal with the characterization of space-time when the matter content is taken to be described in terms of quantum fields.

2 The general setting

The first simplification we will be using is justified by the fact that, despite the many important and sometimes spectacular advances, at this point in time we still do not have a fully workable and completely satisfactory theory of quantum gravity. For instance, we do not know how to construct a quantum state representing Minkowski space-time. In fact, it is well known that any canonical approach to quantum gravity inevitably leads to a timeless theory where the recovery of fully covariant space-time notions becomes, by itself, a nontrivial task. Those approaches usually require selecting a physical observable to play the role of a clock, and while this can be achieved quite satisfactorily in certain cases, the resolution of the problem in full generality is not available.

Therefore, although we will adhere to the view that the fundamental description of everything, including space-time, ought to be always quantum mechanical, we will be using a classical description of the space-time metric.

In general, we will view the so called *classical regimes* in connection with some physical variable as those where such quantities can be described to a sufficient accuracy by their classical counterparts representing the corresponding quantum expectation values. The paradigmatic example here is provided by the coherent states of a harmonic oscillator which correspond to minimal wave-packets with expectation values of position and momentum following the classical equations of motion. In the specific case of the space-time, we will accept that, at the fundamental level it would have a quantum description in terms of some unspecified variables (they might be those of loop quantum gravity (LQG), the “causal set” approach or the “dynamical triangulations” approach, etc) but we will be characterizing them, according to this view, using effectively classical terms. We might consider such description in analogy with the hydrodynamical characterization of a fluid: as representing a good enough description at certain scales, but having a radically different description at a more fundamental level. Einstein’s equations would correspond to the Navier-Stokes equations, the space-time metric to the fluid velocity and density fields, and the atomic and molecular characterization of the matter making the fluid would correspond to the fundamental degrees of freedom of quantum gravity.

According to this view, Einstein’s equations would be of limited validity and there would be conditions where they will not hold. In fact in such situations one can expect a more general failure of the characterization of the situation in terms

of a space-time metric, just as, in the case of a fluid, one might expect not only the violation of Navier-Stokes equations but also of the hydrodynamic characterization of matter, when something like a phase transition from liquid to gas is taking place. There is one example coming from LQG where one faces precisely this kind of breakdown: It correspond to the situation where a collapsing star forms a black hole that eventually evaporates completely [5]. In that work it is argued that although the classical singularity would be resolved in terms of the LQG degrees of freedom, the region of space-time corresponding to the singularity is characterized, in terms of the fundamental variables, by a situation that has no metric counterpart.

On the other hand, the situations I want to consider are those that require a full quantum treatment of the matter fields. For this, we will rely on the standard quantum field theory in curved space-time treatments such as described for instance in [6]. The setting is therefore that of semiclassical gravity where Einstein's equations read:

$$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle \quad (1)$$

and matter is described in terms of states of a quantum field, which, in our case, will be the scalar field of inflationary cosmology.

The validity of such semiclassical treatment is, as we have indicated, of limited scope, and would require, among other things, the scalar curvature of space-time to be small compared to l_p^{-2} (l_p is the Planck length). The inflationary regime is thought to be associated to scales that are way below the Planck mass and thus this part of the requirement should be easily satisfied in the case we want to consider.

3 The Stern-Gerlach analogy

The general issue we want to consider is the emergence of the seeds of quantum structure from the quantum fluctuations of the inflaton vacuum. The aspect that will be guiding our inquest is the required change in symmetry between the conditions characterized by a classical and unperturbed Robertson Walker space-time, where the inflaton field's zero mode is slowly rolling down the potential, and where the other modes are in the the Bunch-Davies vacuum, to a stage characterized by a slightly perturbed RW space-time containing small anisotropies and inhomogeneities, and a quantum state of the inflaton where the expectation value of the energy momentum tensor has the corresponding anisotropies and inhomogeneities.

The question is how to characterize the evolution in time (because, after all, *emergence* is a word that has very clear time connotations¹), from a situation corresponding to a homogeneous and isotropic (H&I) background and a quantum aspect characterized by a H&I state, to a stage lacking such symmetries. This issue has been the central focus of the discussion in previous works: Is there a measurement involved? Can we account for it using just decoherence? Should we rely on the many worlds interpretation (MWI) or must we call upon a novel gravity-induced collapse

¹ Something emerges when it is not present at a certain time but it is present at a later time.

of the wave function following the ideas of Penrose and Diósi? I have extensively argued that among those the only option is the last one, however, here I will focus on aspects that should be dealt with, even if one is intent on sticking with some of the alternative views mentioned above.

In order to clarify the issue, I shall consider a much simpler problem, and point to the parallels with the inflationary problem, as well as to those aspects where the analogy breaks down.

Consider a standard Stern-Gerlach experiment: The setting involves an electron that has been prepared moving along the x axis from the $x < 0$ region, towards a magnet placed at the origin of coordinates. The inhomogeneous magnetic field points along the y axis so that the electron will be diverted towards the $+y$ or $-y$ directions depending on whether the spin state of the electron is $|+\rangle$ or $|-\rangle$ (we are taking the basis to be that of eigen-states of the spin along the y axis). If the spin was prepared initially in the direction $+x$ (eigenstate of the x component of the spin) we know that there is a 50% probability that the electron will be diverted towards the $+y$ direction.

Let us imagine for a moment that we do not fully understand the theory, that there are aspects of the electromagnetic interaction that still elude us (the allegory here alludes to the quantum theory of gravity of course). Now let us consider the theoretical analysis of the said experiment: If we do not invoke any sort of reduction, or collapse of the wave function, the result of the unitary evolution will be a state that corresponds to a superposition of the electron going up and the electron going down. However suppose we want to investigate in depth what happens when the electron is deflected: Say, we want to understand exactly the details of the momentum transferred from the magnetic field to the electron. We could, for instance, find that the momentum transfer has components predominantly in the Y direction (depending on the deflection, with a sign that depends on the alternative) but is accompanied by momentum components in the X direction (of a specific and correlated magnitude), and use this to study the change in the kinetic energy. We can even inquire about the rate of transfer as the electron moves along the X direction (by considering an appropriate wave packet characterization of the electron and, say, following the expectation value of the center of mass). How can we do that if we maintain that the electron is, even after the passing through the magnet, in the superposition of moving up and moving down? In that case, if we try to compute the momentum transfer from the EM field to the electron we will find that it is zero.

Suppose we want to further inquire about the back reaction of the electron on the EM field. It would seem very difficult to do so without incorporating the collapse. Note that, if we are so inclined, we could even adopt the many worlds interpretation (MWI) but still concern ourselves with one of the realizations of the electron's path, that which corresponds to "our branch of the many worlds". Now, suppose we wanted to do this before we had a fully workable quantum theory of the electromagnetic field, but instead we had very refined experimental data about the back reaction acting on the magnet as a result of the electron scattering. Could we not hope to investigate some of the properties of the quantum electromagnetic field using a combination of the data, some rough classical characterization of the EM field,

taken to be only valid to a certain degree (evidently not in the full description of the back reaction, but, perhaps, as it applies to the “after” and “before” state of the EM field)? Could we hope to do that if we had never even been able to consider the back reaction, as a result of our failure to acknowledge that the full superposition (in which the expectation value of the momentum transfer was zero) was not the appropriate description? It seems we could start considering something like the dispersion of the momentum transfer but, that would be very difficult due to our lacking of a workable quantum theory of Maxwell’s field.

The situation we face regarding the problem of the emergence of seeds of structure in inflationary cosmology is, in a sense, analogous to the one above: the symmetry of homogeneity and isotropy in the cosmological case has in the example above a simple counterpart: the symmetry $y \rightarrow -y$.

The most important aspect where the analogy breaks down is the fact that in contrast with the Stern-Gerlach example above, in cosmology we can not call upon external observers, and, what is even worse, the emergence of structure, which is what we want to explain, is a prerequisite for the subsequent emergence of anything one can consider as an “observer”.

The issue of symmetry is brought in because, as we all know, when dealing with complicated problems, symmetry arguments are often one of the few paths available and arrive to clear and definite conclusions, and thus they provide the only hope to make progress.

4 A Word About Collapse Theories

The idea of modifying quantum theory by adding to it a mechanism for explicit dynamical reduction has a long history. The existing work in this direction includes: GRW [7], Pearle [8], Diósi [2], Penrose [1], Bassi [9] (where it’s worthwhile noting some recent advances towards making collapse theories compatible with special relativity [10, 11]), and recently Weinberg [12].

As an example, let us consider the modification of the Schrödinger equation that underlies the Continuous Spontaneous Localization (CSL) theory, developed in [13]:

$$d|\psi\rangle = -\left\{i\hat{H} - \frac{\lambda^2}{2}(\hat{A} - \langle\psi|\hat{A}|\psi\rangle)^2\right\}dt + \lambda(\hat{A} - \langle\psi|\hat{A}|\psi\rangle)dW_t\}|\psi\rangle, \quad (2)$$

where W_t is a Wiener process ($\overline{W_t^2} = t$).

Its merit is that it includes the unitary Schrödinger evolution U and the non-deterministic, non-unitary reduction process R (for measuring \hat{A}) in a unified fashion. The proposal for particles assumes that the fundamental localization takes place in position or configuration space, thus $\hat{A} = \hat{\mathbf{X}}$. The value of the parameter λ is taken to be small enough so that particle physics is not strongly affected, but large enough so that it leads to a rapid localization of macroscopic objects.

In the remainder of this manuscript, we will not follow any of those proposals, but take from them only some essential aspects. What we want to do, at this point, is present a generic formalism capable of treating the cosmological problem that motivates this line of research, within the context of semiclassical gravity with a full quantum treatment for the inflaton field (including its zero mode), incorporating a collapse process. On the other hand, we should mention the work [14], where precisely a version of CSL has been adopted to this problem, as well as an ongoing research project by our group in which a different implementation of those ideas is studied [15].

5 The Self Consistent Semiclassical Configurations

In this section, we will describe the precise formalism that we consider as appropriate to characterize, at the desired level, the situation we will be studying. As we said, the setting is that of semiclassical gravity and quantum field theory in curved space-time. Such setting is often considered as a context in which the space time is fixed and given, and where the quantum fields can, at most, produce some small modifications which are referred to as the back reaction of space-time to the effects of the quantum fields. That point of view will not be sufficient for our purposes, as we want to be able to, in principle, treat the case where the only matter content is represented by the inflaton field and where the space-time is fundamentally tied to its properties: Inflation is supposed to be the result of the non vanishing value of the inflaton potential. In such situation the field is generically treated at a classical level, and only its perturbations are quantized. We want to be able to explore the setting in which the classical quantum partition in the description is, in principle, not tied to a perturbative treatment².

We have formalized these ideas in [16] based on the notion of the ‘‘Self Consistent Semiclassical Configurations’’ (SSC) provided by the following,

Definition 1. The set $g_{\mu\nu}(x), \hat{\phi}(x), \hat{\pi}(x), \mathcal{H}, |\xi\rangle \in \mathcal{H}$ represents a SSC if and only if $\hat{\phi}(x)$ and $\hat{\pi}(x)$ correspond to quantum field operators over the Hilbert space and \mathcal{H} is constructed according to the standard QFT over the curved space-time with metric $g_{\mu\nu}(x)$ (as described in, say [6]), and the state $|\xi\rangle$ in \mathcal{H} is such that:

$$G_{\mu\nu}[g(x)] = 8\pi G \langle \xi | \hat{T}_{\mu\nu}[g(x), \hat{\phi}(x), \hat{\pi}(x)] | \xi \rangle \quad (3)$$

for all the points x in the space-time manifold.

² The point is that a perturbative treatment should be considered as an approximation to something else, and it is very useful when one can establish explicitly what is that which the perturbative treatment is approximately trying to describe.

It is, in a sense, the GR version of Schrödinger-Newton equation [17] where one considers the Schrödinger equation for the wave function ψ of a particle subject to the gravitational interaction described in terms of a Newtonian potential Φ_N :

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2M}\nabla^2\psi + M\Phi_N\psi, \quad (4)$$

where the wave function of the particle is taken as a gravitating mass distribution, therefore

$$\nabla^2\Phi_N = 4\pi GM|\psi|^2. \quad (5)$$

The non linearity implied by these equations is known to lead to interesting and suggestive behavior [18].

6 Collapse

The point, however, is that this setting will not, by itself, be enough to describe the situations involving a collapse of the wave function. As we have argued, in order to be able to describe the evolution in time from an early inflationary era characterized by an H&I situation to a later regime characterized by a situation that is not H&I, we will be relying on the collapse of the wave function, represented here in the simplest fashion: an instantaneous jump in the state of the quantum field.

The collapse process reflecting some remanent signature from a fundamental quantum gravity regime (as suggested by Penrose and Doisi's ideas) will be described here as an instantaneous jump: i.e., besides the standard smooth unitary evolution of a quantum field characterized by the Schrödinger's dynamics (the so called U process, which in the QFT theory setting we are considering here, is usually incorporated using the "Heisenberg picture"), there are, sometimes, spontaneous jumps in the quantum state:

$$\dots|0\rangle_{k_1} \otimes |0\rangle_{k_2} \otimes |0\rangle_{k_3} \otimes \dots \rightarrow \dots|\mathcal{E}\rangle_{k_1} \otimes |0\rangle_{k_2} \otimes |0\rangle_{k_3} \otimes \dots \quad (6)$$

We might view the collapse as triggered by an aspect of the dynamics which is not susceptible of description in standard Hamiltonian terms, but which is nonetheless taken as an interaction treated here in the interaction picture. That is, we take the Hamiltonian part of the evolution and absorb it in the quantum field operators as in the standard Heisenberg picture, but we leave the reminder, viewed as an interaction, to be treated using the interaction picture.

7 Relation to other approaches

This setting might seem very novel and unusual, however, the fact is that it can be seen to lie, unsuspectedly, underneath some more conventional approaches, such as the stochastic gravity formalism [19].

In order to see this, let us consider one of such jumps or collapses: $|\psi(t)\rangle = \theta(t_0 - t)|0\rangle + \theta(t - t_0)|\xi\rangle$, and its gravitational effects.

Now Einstein's semiclassical equations read:

$$G_{\mu\nu} = 8\pi G \langle \psi(t) | \hat{T}_{\mu\nu} | \psi(t) \rangle \quad (7)$$

which we can write as:

$$G_{\mu\nu} = 8\pi G \langle 0 | \hat{T}_{\mu\nu} | 0 \rangle + 8\pi G \xi_{\mu\nu}, \quad (8)$$

where

$$\xi_{\mu\nu} \equiv \theta(t - t_0) (\langle \xi | \hat{T}_{\mu\nu} | \xi \rangle - \langle 0 | \hat{T}_{\mu\nu} | 0 \rangle) \quad (9)$$

might be seen as corresponding to an individual stochastic step. Stochastic gravity might correspond to a continuous version of dynamical collapses (like CSL).

Note: the equation can not be valid on the jump, but might well be so before and after. We take the view, motivated in part by the black hole singularity example in LQG, that during the jump the degrees of freedom of the quantum space-time are excited. In the fluid analogy, this might be thought as corresponding to some chemical reaction or phase transition occurring in the fluid. It is clear that during such processes, which generally involve energy flux between the atomic or molecular degrees of freedom to the macroscopic degrees of freedom characterized in terms of the fluid variables, the Navier-Stokes equations can not be valid. If, however, the phase transition takes place rapidly one can assume such equation to be valid before and after the chemical reaction or phase transition.

Next, consider the inflationary problem at hand, and assume one adopts one of the more popular postures regarding the emergence of classicality, or more precisely the generation of primordial inhomogeneities and anisotropies. These include for instance i) the notion that after a given mode exits the horizon (its physical wavelength as seen in a co-moving frame, becomes larger than the Hubble radius) the fluctuation corresponding to that mode becomes classical, or ii) that, due to some decoherence effect, we can at a certain point adopt the Many Worlds Interpretation of quantum theory, and consider the state of the quantum field as characterizing not our universe, but an ensemble of universes of which ours is just a typical element. Now, let us say that in case i) we want to produce a description (even an approximate one) of our universe concentrating, for simplicity, on a single mode, but we want a description that is valid before the mode exits the horizon and afterwards. In that case, the approach I will present, seems to be the best one can do, as long as we do not have a workable theory of quantum gravity which allows us to characterize space-time in a full quantum language. In case ii) we might also be interested in putting together the characterization of our space-time before the decoherence is taken to

be effective and the one describing the particular branch of the many worlds, or the particular element of the enabled universes in which we happen to find ourselves. Again in that situation the analysis I will present would offer perhaps the furthest one can go in achieving the said goal, given the present stage of the development of candidate theories of quantum gravity.

8 Application to Inflation

As we discussed in the previous sections, space-time will be treated as classical. In the case of interest, working in a specific gauge, and ignoring the tensor perturbations the metric is taken as:

$$ds^2 = a^2(\eta) [-(1 + 2\psi)d\eta^2 + (1 - 2\psi)\delta_{ij}dx^i dx^j], \psi(\eta, \mathbf{x}) \ll 1 \quad (10)$$

with $a(\eta)$ the scale factor and $\psi(\eta, x)$ representing (to the extent that it is nonzero) a possible slight departure from homogeneity and isotropy of the space-time, the so called Newtonian potential. We will use the notation $\mathcal{H} \equiv a^{-1}da/d\eta$ (not to be confused with the hatted quantity that stands for a Hilbert space).

Also, as explained before, the scalar field, which we take here to be described by the simple action $S = 1/2 \int d^4x (\nabla_\mu \phi \nabla^\mu \phi - m^2 \phi^2)$, including the zero mode (which in standard discussions of inflation is usually treated at a classical level) is treated here using quantum field theory on curved space times, so we write:

$$\hat{\phi}(x) = \sum_{\alpha} (\hat{a}_{\alpha} u_{\alpha}(x) + \hat{a}_{\alpha}^{\dagger} u_{\alpha}^*(x)), \quad (11)$$

with the functions $u_{\alpha}(x)$ a complete set of normal modes, orthonormal w.r.t. the symplectic product:

$$((\phi_1, \pi_1), (\phi_2, \pi_2))_{\text{Sympl}} \equiv -i \int_{\Sigma} [\phi_1 \pi_2^* - \pi_1 \phi_2^*] d^3x. \quad (12)$$

For simplicity, we set the problem in a co-moving coordinate box of size L .

Finally, one constructs the state such that Einstein semiclassical equations hold. This is nontrivial, but it is a well defined problem. In what follows, the discussion will omit some complications that are required for the rigorous analysis and discussed in detail in [16], but which are not central to the issue at hand. They have to do with the hermiticity of the operators that play a central role in the collapse, an issue that will be overlooked here to avoid nonessential complications in the presentation.

The Homogeneous and Isotropic case: SSC I

We assume an almost de Sitter slow-roll expansion characterized by the parameters $H_0^{(I)}$ and $\varepsilon^{(I)}$ (using standard inflationary notation [20]).

The quantum field theory construction requires a complete set of modes, which we take to be of the form $u_{\mathbf{k}}^{(I)}(x) = v_{\mathbf{k}}^{(I)}(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}/L^{3/2}$.

The field equation of motion then leads to:

$$\ddot{v}_{\mathbf{k}}^{(I)} + 2\mathcal{H}^{(I)}\dot{v}_{\mathbf{k}}^{(I)} + \left(k^2 + a^{2(I)}m^2\right)v_{\mathbf{k}}^{(I)} = 0, \quad (13)$$

for modes, which must be normalized according to

$$v_{\mathbf{k}}^{(I)}v_{\mathbf{k}}^{(I)*} - \dot{v}_{\mathbf{k}}^{(I)}\dot{v}_{\mathbf{k}}^{(I)*} = i\hbar a^{-2(I)}. \quad (14)$$

For the modes with $k \neq 0$, the most general solution to the evolution equation is a linear combination of the functions: $\eta^{3/2}H_\nu^{(1)}(-k\eta)$ and $\eta^{3/2}H_\nu^{(2)}(-k\eta)$, (the Hankel functions of first and second kind), with $(\nu^{(I)})^2 = (9/4) - (m/H_0^{(I)})^2$. However, these functions are not well behaved at the origin and thus the zero mode is not included. For $k = 0$ the general solution to the equation is a linear combination of the functions $\eta^{(3-2\nu)/2}$ and $\eta^{(3+2\nu)/2}$. The choice can be made arbitrarily provided it has a positive symplectic norm. We take:

$$v_0^{(I)}(\eta) = \sqrt{\frac{\hbar}{H_0^{(I)}}} \left[1 - \frac{i}{6} \left(-H_0^{(I)}\eta\right)^3 \right] \left(-H_0^{(I)}\eta\right)^{m^2/3H_0^{2(I)}}. \quad (15)$$

For the $k \neq 0$ modes, we make the Bunch-Davies choice: i.e., we use modes that, in the asymptotic past, behave as purely ‘‘positive frequency solutions’’. This fixes $\mathcal{H}^{(I)}$ as the Fock space of the SSC-I construction.

To complete the SSC construction we still need to find a state $|\xi^{(I)}\rangle \in \mathcal{H}^{(I)}$ such that its expectation value for the energy-momentum tensor leads to the desired nearly de Sitter expansion. Consider a state in which all the modes with $k \neq 0$ are in their vacuum state, while the zero mode is excited in a coherent state:

$$|\xi^{(I)}\rangle = c e^{\xi_0^{(I)}\hat{a}_0^{(I)\dagger}} |0^{(I)}\rangle, \quad (16)$$

Using Einstein’s equations for the metric with a vanishing Newtonian potential, and the fact that for a coherent state we have

$$\langle \xi^{(I)} | : (\phi^{(I)})^2 : | \xi^{(I)} \rangle = (\langle \xi^{(I)} | (\phi^{(I)}) | \xi^{(I)} \rangle)^2, \quad (17)$$

one finds that the expectation value of the field should satisfy:

$$\langle \xi^{(I)} | (\phi^{(I)}) | \xi^{(I)} \rangle \propto \eta \sqrt{\varepsilon^{(I)} m^2 / 3 (H_0^{(I)})^2}. \quad (18)$$

On the other hand taking the parameter $\xi_0^{(I)}$ as real we find:

$$\langle \xi^{(I)} | \hat{\phi}^{(I)}(x) | \xi^{(I)} \rangle = \frac{2\xi_0^{(I)}}{L^{3/2}} \sqrt{\frac{\hbar}{H_0^{(I)}}} \left(-H_0^{(I)} \eta \right)^{m^2/3H_0^{2(I)}}. \quad (19)$$

That is, we will have compatibility if we set:

$$\varepsilon^{(I)} = \frac{m^2}{3H_0^{2(I)}}, \quad H_0^{(I)} = 16\pi G\hbar\varepsilon^{(I)} \frac{(\xi_0^{(I)})^2}{L^3}. \quad (20)$$

This completes the explicit SSC-I construction representing an H&I state and space-time metric, corresponding to the early stages of inflation.

Next, we want to consider a situation where the universe is no longer H&I but has been excited in the \mathbf{k}_0 mode: We will denote this new SSC by SSC-II.

A simple inhomogeneous and anisotropic case: SSC II

It will be characterized by the parameters $H_0^{(II)}$ and $\varepsilon^{(II)}$ (which might, in principle, differ slightly from those corresponding to the SSC-I discussed in the previous section), and a Newtonian potential described by an (in principle) arbitrary function $\psi(\eta, \mathbf{x}) = \varepsilon P(\eta) \text{Cos}(\mathbf{k}_0 \cdot \mathbf{x})$, where $P(\eta)$ is an (in principle) arbitrary function, and ε is a small (expansion) parameter (please do not confuse with ε).

The strategy: We first construct the “generic” Hilbert space assuming that $P(\eta)$ is given. Then, make an “educated” guess for the form of the quantum state, and by requiring that our construction be a SSC we will find what the function $P(\eta)$ ought to be.

The first step is to find the complete set of modes, which we write as:

$$u_{\mathbf{k}}^{(II)}(x) = \frac{1}{L^{3/2}} [v_{\mathbf{k}}^{(II)0}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \varepsilon(\delta v_{\mathbf{k}}^{(II)-}(\eta) e^{i(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{x}} + \delta v_{\mathbf{k}}^{(II)+}(\eta) e^{i(\mathbf{k}+\mathbf{k}_0)\cdot\mathbf{x}})] \quad (21)$$

to the zeroth order in ε , the evolution equation is given by

$$\ddot{v}_{\mathbf{k}}^{(II)0} + 2\mathcal{H}^{(II)}\dot{v}_{\mathbf{k}}^{(II)0} + (k^2 + a^{2(II)}m^2)v_{\mathbf{k}}^{(II)0} = 0, \quad (22)$$

with normalization condition

$$v_{\mathbf{k}}^{(II)0} \dot{v}_{\mathbf{k}}^{(II)0*} - \dot{v}_{\mathbf{k}}^{(II)0} v_{\mathbf{k}}^{(II)0*} = i\hbar a^{-2(I)}, \quad (23)$$

which is identical to the construction we have already done. Thus, we take the $v_{\mathbf{k}}^{(II)0}(\eta)$ as before.

At first order in ε the corresponding evolution equation takes the form

$$\delta \ddot{v}_{\mathbf{k}}^{(II)\pm} + 2\mathcal{H}^{(II)}\delta \dot{v}_{\mathbf{k}}^{(II)\pm} + \left[(\mathbf{k} \pm \mathbf{k}_0)^2 + a^{2(II)}m^2 \right] \delta v_{\mathbf{k}}^{(II)\pm} = F_{\mathbf{k}}(\eta) \quad (24)$$

where

$$F_{\mathbf{k}}(\eta) \equiv 4\dot{v}_{\mathbf{k}}^{(\text{II})0} \dot{P} - 2 \left(2k^2 + a^{2(\text{II})} m^2 \right) v_{\mathbf{k}}^{(\text{II})0} P. \quad (25)$$

The normalization condition (needed only at one time) is:

$$\dot{v}_{\mathbf{k}+\mathbf{k}_0}^{(\text{II})0*} \delta v_{\mathbf{k}}^{(\text{II})+} - v_{\mathbf{k}+\mathbf{k}_0}^{(\text{II})0*} \delta \dot{v}_{\mathbf{k}}^{(\text{II})+} - \dot{v}_{\mathbf{k}}^{(\text{II})0} \delta v_{\mathbf{k}+\mathbf{k}_0}^{(\text{II})-*} + v_{\mathbf{k}}^{(\text{II})0} \delta \dot{v}_{\mathbf{k}+\mathbf{k}_0}^{(\text{II})-*} \quad (26)$$

$$= 4 \left(v_{\mathbf{k}}^{(\text{II})0} \dot{v}_{\mathbf{k}+\mathbf{k}_0}^{(\text{II})0*} - \dot{v}_{\mathbf{k}}^{(\text{II})0} v_{\mathbf{k}+\mathbf{k}_0}^{(\text{II})0*} \right) P. \quad (27)$$

If we had $P(\eta)$ and the initial conditions for the $\delta v_{\mathbf{k}}$, the equation above would define a unique solution. As we said, we will assume that $P(\eta)$ is given and take the initial conditions to be

$$\delta v_{\mathbf{k}}^{(\text{II})\pm}(\eta_c) = 0, \quad \delta \dot{v}_{\mathbf{k}}^{(\text{II})\pm}(\eta_c) = 4v_{\mathbf{k}}^{(\text{II})0}(\eta_c)P(\eta_c). \quad (28)$$

This finishes the generic (i.e. for arbitrary P) construction of $\hat{\mathcal{H}}^{(\text{II})}$.

Next we need to find the state $|\zeta^{(\text{II})}\rangle \in \mathcal{H}^{(\text{II})}$ that completes the SSC-II construction. The symmetries of the space-time led us to consider the ‘‘ansatz’’:

$$|\zeta^{(\text{II})}\rangle = \dots |\zeta_{-2\mathbf{k}_0}^{(\text{II})}\rangle \otimes |\zeta_{-\mathbf{k}_0}^{(\text{II})}\rangle \otimes |\zeta_0^{(\text{II})}\rangle \otimes |\zeta_{\mathbf{k}_0}^{(\text{II})}\rangle \otimes |\zeta_{2\mathbf{k}_0}^{(\text{II})}\rangle \dots \quad (29)$$

The vector in Fock space is characterized by the specific modes that are excited (all other modes are assumed to be in the vacuum of the corresponding oscillator) and the parameters $\zeta_{\mathbf{k}}^{(\text{II})}$ indicate the coherent state for the mode \mathbf{k} .

The expectation value of the field operator in such a state is given by

$$\phi_{\zeta}^{(\text{II})}(x) = \phi_{\zeta,0}^{(\text{II})}(\eta) + \left(\delta \phi_{\zeta,\mathbf{k}_0}^{(\text{II})}(\eta) e^{i\mathbf{k}_0 \cdot \mathbf{x}} \right) + \left(\delta \phi_{\zeta,2\mathbf{k}_0}^{(\text{II})}(\eta) e^{i2\mathbf{k}_0 \cdot \mathbf{x}} \right) + \dots \quad (30)$$

We note that the coefficients $\delta \phi_{\zeta,n\mathbf{k}_0}^{(\text{II})}(\eta)$ have a contribution from the modes $n\mathbf{k}$, $(n-1)\mathbf{k}$ and $(n+1)\mathbf{k}$. We set $\delta \phi_{\zeta,n\mathbf{k}_0}^{(\text{II})}(\eta) = 0$ for all $n \geq 2$, simply by imposing the required relations between the parameters $\zeta_{\pm\mathbf{k}_0}^{(\text{II})}$, $\zeta_{\pm 2\mathbf{k}_0}^{(\text{II})}$, $\zeta_{\pm 3\mathbf{k}_0}^{(\text{II})}$, etc. It is easy to see that $|\zeta_{\pm n\mathbf{k}_0}^{(\text{II})}\rangle \sim \epsilon^n |\zeta_0^{(\text{II})}\rangle$.

The conditions above ensure that there are no terms in $e^{\pm i n \mathbf{k}_0 \cdot \mathbf{x}}$ (with $n \geq 2$) appearing in the expectation value of the energy-momentum tensor. That is necessary for the compatibility of our state ansatz with the semiclassical Einstein’s equations. We studied these in detail up to to the first order in ϵ .

The zero order equations are identical to those we found in constructing the SSC-I. They fix the construction of SSC to the lowest order, i.e. they determine the relation between $a^{(\text{II})}$ and $\zeta_0^{(\text{II})}$.

Considering the next order one obtains after a lengthy calculation the key result, that enables us to carry out the construction in a complete manner: That the equations can be combined into a simple dynamical equation for the Newtonian potential, which is independent of the first order quantities and where, at the level of precision we are working at, the equation above becomes simply:

$$\ddot{P} + \varepsilon^{(\text{II})} \mathcal{H}^{(\text{II})} \dot{P} + \left[k_0^2 - \varepsilon^{(\text{II})} \mathcal{H}^{2(\text{II})} \right] P = 0. \quad (31)$$

The general solution

$$P(\eta) = C_1 \eta^{\frac{1}{2}[1+\varepsilon^{(\text{II})}]} J_\alpha(-k\eta) + C_2 \eta^{\frac{1}{2}[1+\varepsilon^{(\text{II})}]} Y_\alpha(-k\eta), \quad (32)$$

where $J_\alpha(-k\eta)$ and $Y_\alpha(-k\eta)$ are the Bessel functions of the first and second kind, $\alpha = [1 + 3\varepsilon^{(\text{II})}]/2$.

Einstein's equations lead, as is well known, to constraints which, at this order provide relations involving the initial values that would determine the specific solution $P(\eta)$:

$$\varepsilon \begin{pmatrix} P \\ \dot{P} \end{pmatrix} = \frac{\sqrt{4\pi G \varepsilon^{(\text{II})} \mathcal{H}^{(\text{II})}}}{k_0^2 - \mathcal{H}^{2(\text{II})} \varepsilon^{(\text{II})}} \times \begin{pmatrix} (3\mathcal{H}^{(\text{II})} - am\sqrt{3/\varepsilon^{(\text{II})}}) & 1 \\ (am\sqrt{3/\varepsilon^{(\text{II})}} \mathcal{H}^{(\text{II})} - k_0^2 + (\varepsilon^{(\text{II})} - 3)\mathcal{H}^{2(\text{II})}) & -\mathcal{H}^{(\text{II})} \end{pmatrix} \cdot \begin{pmatrix} \delta\phi_{\zeta, \mathbf{k}_0}^{(\text{II})} \\ \delta\dot{\phi}_{\zeta, \mathbf{k}_0}^{(\text{II})} \end{pmatrix}. \quad (33)$$

Thus given $\delta\phi_{\zeta, \mathbf{k}_0}^{(\text{II})}(\eta_c)$ and $\delta\dot{\phi}_{\zeta, \mathbf{k}_0}^{(\text{II})}(\eta_c)$, we have a completely determined space-time metric. In particular, we have a completely determined function $P(\eta)$ and thus, as discussed around equation (26), a completely specified set of mode functions for the expansion of the field operator. Furthermore, those determine the state parameters $\zeta_{\mathbf{k}_0}$ (and thus the rest as well). Thus, we have a complete SSC-II (to this order in ε).

The collapse: Joining SSC-I and SSC-II

. Next, we want to consider a space-time that includes a collapse. That is, a space-time that results from the *the matching* of the two constructions. We will consider here that the collapse corresponds to a hypersurface that is matched to the hypersurfaces $\eta = \eta_c$ of SSC-I and SSC-II. Note that this gives such hypersurface Σ_c a preferred status in the resulting space-time, and is not something to be thought as related to a gauge freedom: To the past of that hypersurface Σ_c the space-time is H&I, and to the future it is not. We will assume here the induced metric is continuous on Σ_c . This requires $P(\eta_c) = 0$ and thus

$$(3\mathcal{H}^{(\text{II})} - am\sqrt{3/\varepsilon^{(\text{II})}}) \delta\phi_{\zeta, \mathbf{k}_0}^{(\text{II})}(\eta_c) + \delta\dot{\phi}_{\zeta, \mathbf{k}_0}^{(\text{II})}(\eta_c) = 0, \quad (34)$$

and therefore

$$\varepsilon \dot{P} = -\sqrt{4\pi G \varepsilon^{(\text{II})} \mathcal{H}^{(\text{II})}} \delta\phi_{\zeta, \mathbf{k}_0}^{(\text{II})}(\eta_c), \quad (35)$$

which indicates a discontinuity in the extrinsic curvature of the hypersurface Σ_c .

Assume that the collapse is characterized by a loose analogy with ‘‘an imprecise measurement’’ (of the operators $\hat{\phi}_{\mathbf{k}_0}^{(\text{I})}(\eta)$) in standard QT: Before the collapse, the operator had zero expectation value but an uncertainty $\Delta \hat{\phi}_{\mathbf{k}_0}^{(\text{I})}(\eta_c)$, and thus we as-

sume that after the collapse, the new expectation value will fall in that range. We thus consider what would be the energy momentum tensor computed using a state that results from such measurement (on the SSC-I side of Σ_c) and demand that the state on the SSC-II side be such that the energy momentum tensor on the SSC-II side of Σ_c be exactly that. The final result is then:

$$\delta\phi_{\zeta_r, \mathbf{k}_0}^{(II)}(\eta_c) = x_{\mathbf{k}_0} \sqrt{\langle 0_{\mathbf{k}_0}^{(I)} | [\Delta \hat{\phi}_{\mathbf{k}_0}^{(I)}(\eta_c)]^2 | 0_{\mathbf{k}_0}^{(I)} \rangle} \approx x_{\mathbf{k}_0} a(\eta_c)^{-1} \sqrt{\frac{\hbar}{2k}}$$

with $x_{\mathbf{k}_0}$ a random number taken from a distribution characterized by a Gaussian function centered at zero with unit-spread. A choice of the random number $x_{\mathbf{k}_0}$ then determines the SSC-II.

Thus, we have a well defined framework where one could, in principle, carry out all the analysis of the collapse approach to the inflationary origin of the seeds of cosmic structure.

9 Phenomenological studies

In a realistic situation, we need to consider a collapse of not just one, but of all the modes. Thus in contrast with the previous analysis, we have adopted for this purpose a simplified treatment that makes the realistic problem manageable. We note that considering simultaneously all modes is required if we want to compare the theory with observations.

In this simplified treatment, one avoids the complications of the previous treatment by ignoring the multiplicity of Hilbert spaces and considering the quantum states that result from collapses to be elements of the Hilbert space based on the initial homogeneous and isotropic space-time (as is done in standard treatments). We thus split the treatment into that of a classical homogeneous (“background”) part and an in-homogeneous part (“fluctuation”), i.e. $g = g_0 + \delta g$, $\phi = \phi_0 + \delta\phi$.

The background is taken again to be Friedmann-Robertson universe (with vanishing Newtonian potential), and the homogeneous scalar field $\phi_0(\eta)$. In the previous, more precise treatment this would have corresponded to the zero mode of the quantum field.

The main difference, with respect to the ordinary approach, will be in the spatially dependent perturbations. Here, our approach indicates we should quantize the scalar field but not the metric perturbation.

We will set $a = 1$ at the “present cosmological time”, and assume that the inflationary regime ends at a value of $\eta = \eta_0$, negative and very small in absolute terms. Again, in our case the semiclassical Einstein’s equations, at lowest order lead to

$$\nabla^2 \Psi = 4\pi G \dot{\phi}_0 \langle \delta\phi \rangle = s \langle \delta\phi \rangle, \quad (36)$$

where $s \equiv 4\pi G \dot{\phi}_0$.

Consider the quantum theory of the field $\delta\phi$. In this practical treatment it is convenient to work with the rescaled field variable $y = a\delta\phi$ and its conjugate momentum $\pi = \delta\dot{\phi}/a$. We decompose the field and momentum operators as:

$$y(\eta, \mathbf{x}) = \frac{1}{L^3} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \hat{y}_k(\eta), \quad \pi_y(\eta, \mathbf{x}) = \frac{1}{L^3} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \hat{\pi}_k(\eta), \quad (37)$$

where $\hat{y}_k(\eta) \equiv y_k(\eta)\hat{a}_k + \bar{y}_k(\eta)\hat{a}_{-k}^\dagger$ and $\hat{\pi}_k(\eta) \equiv g_k(\eta)\hat{a}_k + \bar{g}_k(\eta)\hat{a}_{-k}^\dagger$. The usual choice of modes $y_k(\eta) = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{\eta k}\right) \exp(-ik\eta)$, $g_k(\eta) = -i\sqrt{\frac{k}{2}} \exp(-ik\eta)$, which leads to what is known as the Bunch-Davies vacuum: the state defined by $\hat{a}_k|0\rangle = 0$. At this point it is worthwhile to remind the reader that this state is translationally and rotationally invariant, as can be easily checked by applying the corresponding rotation and displacement operators to it. Note also that $\langle 0|\hat{y}_k(\eta)|0\rangle = 0$ and $\langle 0|\hat{\pi}_k(\eta)|0\rangle = 0$. The collapse will modify the state and thus expectation values of the operators $\hat{y}_k(\eta)$ and $\hat{\pi}_k(\eta)$.

Next, we specify the rules according to which the collapse happens, and thus the state $|\Theta\rangle$ after the collapse. We assume that after the collapse, the expectation values of the field and momentum operators in each mode will be related to the uncertainties of the pre-collapse state (these quantities for the vacuum are *not* zero).

In the vacuum state, \hat{y}_k and $\hat{\pi}_k$ are characterized by Gaussian wave functions centered at 0 with spread Δy_k and $\Delta \pi_{y_k}$, respectively.

We will want to consider various possibilities for the detailed form of this collapse. Thus, for their generic form, associated with the ideas above, we assume that at time η_k^c the part of the state corresponding to the mode \mathbf{k} undergoes a sudden jump so that, immediately afterwards, the state describing the system is such that

$$\langle \hat{y}_k(\eta_k^c) \rangle_\Theta = x_{k,1} \sqrt{\Delta \hat{y}_k}, \quad \langle \hat{\pi}_k(\eta_k^c) \rangle_\Theta = x_{k,2} \sqrt{\Delta \hat{\pi}_k^y}, \quad (38)$$

where $x_{k,1}, x_{k,2}$ are (single specific values) selected randomly from within a Gaussian distribution centered at zero with spread one.

Finally, using the evolution equations for the expectation values (i.e. using Ehrenfest's Theorem), we obtain $\langle \hat{y}_k(\eta) \rangle$ and $\langle \hat{\pi}_k(\eta) \rangle$ for the state that resulted from the collapse for all later times.

Analysis of the phenomenology

The semi-classical version of the perturbed Einstein's equation that, in our case, leads to $\nabla^2 \Psi = 4\pi G \dot{\phi}_0 \langle \delta\phi \rangle$ indicates that the Fourier components at the conformal time η are given by:

$$\Psi_k(\eta) = -(s/ak^2) \langle \hat{\pi}_k(\eta) \rangle. \quad (39)$$

Prior to the collapse, the state is the BD vacuum, and it is easy to see that $\langle 0|\hat{\pi}_k(\eta)|0\rangle = 0$, so in that situation we would have $\Psi_k(\eta) = 0$. However, after the collapse has occurred, we have instead: $\Psi_k(\eta) = -(s/ak^2) \langle \Theta|\hat{\pi}_k(\eta)|\Theta\rangle \neq 0$.

From those quantities, we can reconstruct the Newtonian potential (for times after the collapse):

$$\Psi(\eta, \mathbf{x}) = \frac{1}{L^3} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \Psi_{\mathbf{k}}(\eta) = \sum_{\mathbf{k}} \frac{sU(k)}{k^2} \sqrt{\frac{\hbar k}{L^3}} \frac{1}{2a} F(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (40)$$

where $F(\mathbf{k})$ contains, besides the random quantities $x_{k,i}, i = 1, 2$, the information about the time at which the collapse of the wave function for the mode \mathbf{k} occurs.

We now focus our attention on the ‘‘Newtonian potential’’ on the surface of last scattering: $\Psi(\eta_D, \mathbf{x}_D)$, where η_D is the conformal time at decoupling and \mathbf{x}_D are co-moving coordinates of points on the last scattering surface corresponding to us as observers. The quantity is identified with the temperature fluctuations on the surface of last scattering. Thus:

$$\alpha_{lm} = \int \Psi(\eta_D, \mathbf{x}_D) Y_{lm}^* d^2\Omega. \quad (41)$$

The factor $U(k)$ is call the transfer function and represents known physics like the acoustic oscillations of the plasma. Now, putting all this together we find,

$$\alpha_{lm} = s \sqrt{\frac{\hbar}{L^3}} \frac{1}{2a} \sum_{\mathbf{k}} \frac{U(k)\sqrt{k}}{k^2} F(\mathbf{k}) 4\pi i^l j_l(|\mathbf{k}|R_D) Y_{lm}(\hat{k}), \quad (42)$$

where $j_l(x)$ is the spherical Bessel function of the first kind, $R_D \equiv \|\mathbf{x}_D\|$, and \hat{k} indicates the direction of the vector \mathbf{k} . Note that in the usual approaches it is impossible to produce an explicit expression for this quantity, other than zero.

Thus α_{lm} is the sum of complex contributions from all the modes, i.e., the equivalent to a two dimensional random walk, whose total displacement corresponds to the observational quantity. We then evaluate the most likely value of such quantity, and then pass to the continuum obtaining:

$$|\alpha_{lm}|_{M.L.}^2 = \frac{s^2 \hbar}{2\pi a^2} \int \frac{U(k)^2 C(k)}{k^4} j_l^2(|\mathbf{k}|R_D) k^3 dk. \quad (43)$$

The function $C(k)$ encodes information contained in $F(k)$. For each model of collapse it has a slightly different functional form.

It turns out that in order to get a reasonable spectrum, we have one single simple option: z_k must be almost independent of k . That is: $\eta_k^c = z/k$.

This result shows that the details of the collapse have observational consequences! In fact, we have

$$C(k) = 1 + \frac{2}{z_k^2} \sin^2 \Delta_k + \frac{1}{z_k} \sin(2\Delta_k), \quad (44)$$

where $\Delta_k = k\eta - z_k$, $z_k = \eta_k^c k$ with η representing the conformal time of observation, and η_k^c the conformal time of collapse of the mode k .

If z_k is independent of k this will not modify the form of the spectrum because these functions become constants. We can consider simple departures from the pattern $\eta_k^c = z/k$, say, assuming $\eta_k^c = A/k + B$. These can now be compared with observations! We have carried out a preliminary exploration [21] considering the departures from the HZ spectrum, and a more detailed analysis [22] incorporating the well understood late time physics (acoustic oscillations, etc.) and comparing directly with the observational data. Those represent the first limits on a collapse model coming from the CMB observations. This analysis can now be used to constrain a more specific version of the collapse theories, particularly those schemes which indicate specific ranges for the collapse times and the specific operators involved in the collapse process.

10 Conclusions

We have argued elsewhere extensively about the need to deal with the fact that the standard accounts for the generation of the primordial seeds of cosmic structure from quantum fluctuations during inflation are not completely satisfactory. In the present work, I have focused on the formal implementation of such ideas, and on the fact that, even if one wanted to ignore the conceptual issues that we have pointed out in previous works, but at the same time, one wanted to consider the possibility of a maximal characterization of our universe, one would be making a similar kind of description as that used in the collapse approach we favor.

We have presented a formalism that allows the incorporation of a collapse process within a semiclassical treatment of gravity interacting with quantum fields. Finally, I have made a brief overview of the phenomenological analysis that relied on a simplified treatment, and which made it manageable to consider the realistic situation involving collapses in all modes of the quantum field.

There is, in fact, an interesting possibility of connection of the ideas presented here to some appearing in the context of the singularity resolution LQG, where one expects a failure of the approach to lead to even an approximate characterization, in terms of classical geometry. In this case we have found a specific kind of breakdown of the space-time description at the collapsing hypersurface: A discontinuity in the extrinsic curvature of such hypersurface. The amount of such discontinuity is related to the details of the collapse. The investigation of this issue in the context of candidate theories for quantum space-time would be very interesting.

We believe that one of the best ways to inquire about the interface of quantum and gravitation is by pushing our attempts to describe space-time in the context where quantum effects become important. The inflationary situation offers us a unique opportunity. In order for us to be able to take full advantage of such window into the unknown, we need to start by recognizing the shortcomings in our current treatments. We trust that the program here outlined represents the first steps in that direction.

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