On Motion of the Magellanic Clouds in the Milky Way Gravitational Field

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Abstract We demonstrate that the cosmological constant substantially influences motion of both Magellanic Clouds in the gravitational field of Milky Way.

1 Introduction

It is usually assumed that for the motion on the scales given by the distance of neighbouring galaxies the influence of the cosmological constant is negligible. We demonstrate that the role of the cosmic repulsion is crucial even on such relatively small scales, using calculations of trajectories and binding mass for Magellanic Clouds.

2 Magellanic Clouds and intial conditions

Large Magellanic Cloud (LMC) and Small Magellanic Cloud (SMC) are dwarf galaxies in the vicinity of Milky Way (MW). We assume both SMC and LMC to be test particles moving (independently) in the MW field, since their mass is substantially smaller than that of the MW and their distance from the visible Galaxy disc is substantially larger than its extension. The Newtonian approach can be used since the GR effects are negligible [1]. Velocity (km/s) and radial vectors (kpc) in galactocentric coordinates are [2] given in Table 1.

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Table 1 LMC and SMC coordinates and velocities.

SMC	x	y	z
x_i	15.3	-36.9	-43.3
v_i	-87± 48	-247± 42	149± 37
LMC	x	y	z
x_i	-0.8	-41.5	-26.9
v_i	-86± 12	-268± 11	252± 16

3 Milky Way gravitational field

The gravitational field of the Milky Way is generated in the standard way by the Galactic disc,

$$U_{disk} = -\frac{\xi G M_{disk}}{\sqrt{x^2+y^2+(k+\sqrt{z^2+b^2})^2}}, \tag{1}$$

the Galactic bulge,

$$U_{bulge} = -\frac{G M_{bulge}}{r+c}, \tag{2}$$

and the CDM halo

$$U_{halo} = v_{halo}^2 \ln(r^2+d^2), \tag{3}$$

where $\xi = 1$, $k = 6.5kpc$, $b = 0.26kpc$, $c = 0.7kpc$. There is $M_{disc} = 5\times10^{10}M_\odot$ and $M_{bulge} = 1.5\times10^{10}M_\odot$. Since $v_{halo} = 114kms^{-1}$, $d = 12kpc$, the logarithmic halo model implies the halo mass formula

$$M_{halo} = \frac{2v_{halo}^2 r^3}{G(r^2+d^2)} \Rightarrow M_{halo}(r=60kpc) = 3.5\times10^{11}M_\odot. \tag{4}$$

4 The role of the Cosmological Constant

Outside the CDM halo we assume that the whole Milky Way halo mass is concentrated in its centre, and its gravitational field is then modelled by the Cosmological Paczynski-Wiita (CPW) potential [3]

$$U_{PN} = -\left(\frac{GM}{r}+\frac{\Lambda c^2}{6}r^2\right)\left(1-\frac{2GM}{c^2 r}-\frac{\Lambda}{3}r^2\right)^{-1} \tag{5}$$

Inside the MW halo, the Newtonian cosmological term

$$U_\Lambda = -\frac{\Lambda c^2}{6} r^2, \quad \Lambda = 1.3 \times 10^{-56} \mathrm{cm}^{-2} \tag{6}$$

is added to the terms representing gravitational field of the disk, bulge and halo. At the static radius, $r_s = [(3GM)/(c^2\Lambda)]^{1/3}$, gravitational attraction and cosmic repulsion are balanced. It gives boundary of gravitationally bound system detached from cosmic expansion.

5 Influence of $\Lambda > 0$ on the MC motion

We integrate the equation of motion both to the time when Λ drives the expansion and Milky Way can be considered a fully developed gravitating system, $z \leq 1 \Rightarrow t \simeq 7.5Gyr$. Trajectories are located in the region detached from the cosmological expansion $\rightarrow$ up to the scale $\simeq Mpc$. Typical trajectories are illustrated in Fig. 1 (for details see [1]).

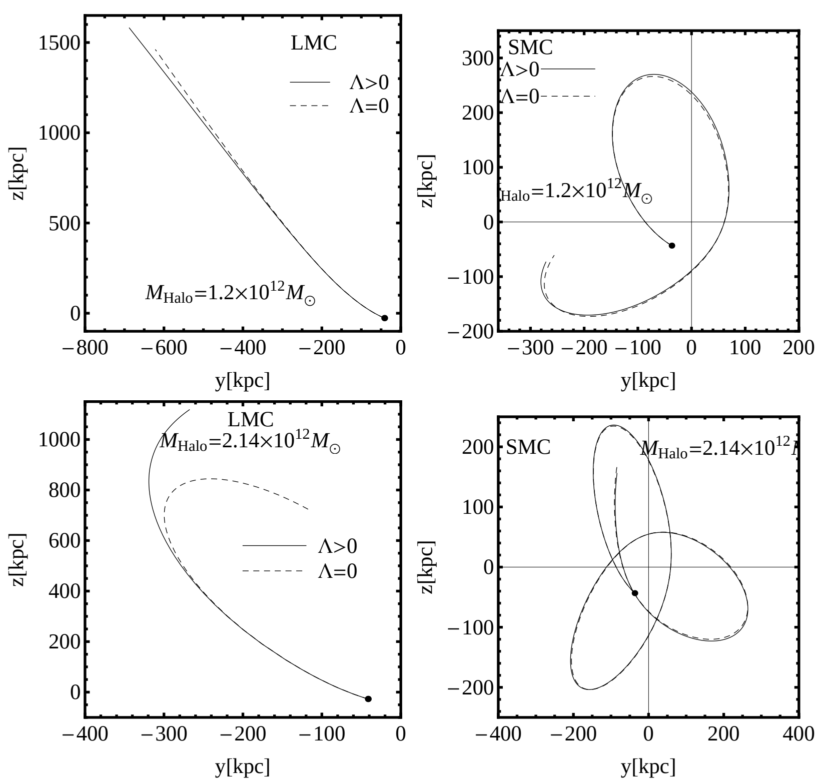

Fig. 1 $Y-Z$ crossections of typical SMC and LMC trajectories.

6 The Binding Mass

Using the effective potential

$$V_{\text{eff}} = \frac{1}{2}\frac{L^2}{r^2} - \frac{GM}{r} - \frac{\Lambda r^2 c^2}{6}, \tag{7}$$

the radius of the unstable circular orbit r_{uc} is given by

$$\frac{\mathrm{d}V_{\text{eff}}}{\mathrm{d}r} = 0 \Rightarrow L^2 = r_{uc}\left(GM - \frac{\Lambda c^2}{3} r_{uc}^2\right). \tag{8}$$

The binding mass M_b is determined by the condition $E(r_0) = E(r_{uc})$.

$$M_b = G^{-1}\left(\frac{1}{r_{uc}} - \frac{1}{r_0}\right)^{-1}\left[\frac{\Lambda c^2}{6}(r_0^2 - r_{uc}^2) + \frac{1}{2}\left(\frac{1}{r_{uc}^2} - \frac{1}{r_0^2}\right) - v_r^2\right] \tag{9}$$

$$L = r_{uc}\left(GM_b - \frac{\Lambda c^2}{3} r_{uc}^2\right) \tag{10}$$

The estimates of the binding mass are in Table 2. Calculations including the halo effect imply M_b at least twice as large. For LMC, influence of $\Lambda > 0$ on the value of M_b is about 50% [4].

Table 2 The dependence of the binding mass M_b and the radius of the unstable circular orbit r_{uc} on the cosmological constant Λ for LMC and SMC.

	SMC	
$\Lambda [cm^{-2}]$	$M_b [10^{11} M_\odot]$	$r_{uc} [kpc]$
1.3×10^{-56}	$6.843(+10,3\%)$	887
0	6.200	∞
	LMC	
$\Lambda [cm^{-2}]$	$M_b [10^{11} M_\odot]$	$r_{uc} [kpc]$
1.3×10^{-56}	$8.863(+7,7\%)$	978
0	8.230	∞

7 Conclusion

Comparing the influence of $\Lambda > 0$ on the MCs motion with those of dynamical friction or gravitational effects of M31, we find the influence of $\Lambda > 0$ to be the same or slightly higher [4].

8 Acknowledgements

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