Quasi-normal Frequencies, Horizon Area Spectra and Multi-horizon Spacetimes

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Abstract We analyse the behavior of the asymptotic frequencies of the spherically symmetric multi-horizon spacetimes (in particular Reissner-Nordström, Schwarzschild-deSitter, Reissner-Nordström-deSitter) and provide some suggestions for how to interpret the results following the spirit of the modified Hod's conjecture. The interpretation suggested is in some sense analogical to the Schwarzschild case, but has some new specific features.

1 Introduction

This short paper refers to work done over a longer period of time contained in papers [1, 2, 3] and also to some extent in [4]. Black holes, when perturbed, show certain characteristic damped oscillations which are called quasi-normal modes (QNMs). The low damped black hole quasi-normal modes are of potential astrophysical interest, as they carry information about the black hole parameters. More than a decade ago it was conjectured that also the highly damped (asymptotic) modes might have physical importance, as they might carry information about quantum black holes [5]. The original conjecture by Hod [5] was later modified by Maggiore [6]. The conjecture of [6] was used in case of Schwarzschild black hole and also in cases of other black holes to derive the area spectra of the black hole horizon. What is still missing is a (unconstrained) application of the conjecture to the most immediate generalizations of Schwarzschild spacetime, to the spherically symmetric multi-horizon spacetimes: Reissner-Nordström (R-N), Schwarzschild-deSitter (S-dS) and Reissner-Nordström-deSitter (R-N-dS) spacetime. This work is trying to fill the gap.

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2 Spherically symmetric multi-horizon black hole spacetimes

The *modified* Hod's (or Maggiore's) conjecture relates asymptotic QNM frequencies ($\omega = \omega_R + i \cdot \omega_I$) to the black hole ADM mass quantum transition as:

$$\Delta M = \lim_{n\to\infty} (\omega_{nI} - \omega_{n-1I}) \doteq \lim_{n\to\infty} \Delta_{(n,n-1)}\omega_{nI}. \tag{1}$$

(For a detailed reasoning see [6]. Also everywhere in the paper we use Planck units and n labels the QNM frequencies monotonically with respect to the imaginary part.) For the multi-horizon spherically symmetric black hole spacetimes (R-N, S-dS, R-N-dS) and the scalar, vector, tensor perturbations the asymptotic QNM frequencies obey the following equation:

$$\sum_{A=1}^{M} C_A \exp\left(\sum_{i=1}^{N} Z_{Ai} \frac{2\pi\omega}{|\kappa_i|}\right) = 0. \tag{2}$$

Z_{Ai} takes one of the values $Z_{Ai} = 0, 1, 2$, furthermore N is the number of horizons and κ_i are the surface gravities of the different horizons. The analytical solutions of formulas (2) are not known and this prevented many people of using the modified Hod's conjecture in R-N, S-dS, R-N-dS cases. In [3] we analysed the behaviour of the solutions of (2) with the following results: If the ratio of all of the surface gravities is a rational number then all the frequencies split in a *finite* number of equispaced families (labeled by a) of the form:

$$\omega_{an} = (\text{offset})_a + in \cdot \text{lcm}(|\kappa_1|, |\kappa_2|, ..., |\kappa_N|). \tag{3}$$

Here lcm is the least common multiple of the numbers in the bracket, hence $\text{lcm}(|\kappa_1|, ..., |\kappa_N|) = p_1|\kappa_1| = ... = p_N|\kappa_N|$, where $\{p_1, ..., p_N\}$ is a set of relatively prime integers. If the ratio of arbitrary two of the surface gravities is irrational, then there does *not* exist an equispaced subsequence in the sequence of asymptotic QNM frequencies. Moreover one can prove [1] for the R-N black hole (but one expects it to hold for all the three cases) that, in case the ratio of the surface gravities is irrational, the $n \to \infty$ limit for $\Delta_{n,n-1}\omega_{nI}$ does *not* exist. Also for the rational ratio of the surface gravities and the R-N black hole the only case in which the limit $n \to \infty$ $\Delta_{n,n-1}\omega_{nI}$ exists is if *all* the frequencies are given by families with the same $(\text{offset})_I$. But this cannot be the case when the ratio of the surface gravities is given by two relatively prime integers whose product is an odd number [1].

The previous considerations seem to suggest, that the modified Hod's conjecture has very little chance to survive the multi-horizon case. However the significantly different behaviour for the cases of rational / irrational ratios of surface gravities and the general splitting of frequencies into families seem to indicate something important. Moreover, it was already observed that surface gravities rational ratios have significant consequences for the multi-horizon spacetime thermodynamics [7]. Based on this observations let us pick the R-N case where the thermodynamical interpretation is straightforward and consider the following: Let us presuppose that both of the horizons in the R-N spacetime, the outer horizon with the area A_+ and the inner Cauchy horizon with the area A_- have equispaced area spectra given as[1] $A_\pm = 8\pi l_p^2 \gamma \cdot n_\pm$.

The perturbations are supposed to carry *no* charge, so one expects that only the ADM mass of the black hole will be changed. Thus [2] one can write the change of the areas of the black hole horizons as:

$$\Delta A_\pm = \frac{8\pi \Delta M}{\kappa_\pm}. \tag{4}$$

But $\Delta A_\pm$ can be given only as $\Delta A_\pm = 8\pi\gamma m_\pm$, which implies $\Delta M = \gamma\kappa_\pm m_\pm$. Furthermore, this implies $m_+\kappa_+ = m_-\kappa_-$ and thus $\kappa_+/\kappa_- = m_-/m_+$. This means that if the single ADM mass transitions have to be allowed the surface gravities ratio must be rational. Furthermore if one wants the emitted mass quantum to be as small as possible, such that it is still compatible with the quantization of the two horizon's areas one obtains:

$$\Delta M = \gamma \cdot \mathrm{lcm}(\kappa_+, |\kappa_-|). \tag{5}$$

Then modified Hod's conjecture suggests that

$$\lim_{n\to\infty} \Delta_{n,n-1}\omega_{nI} = \gamma \cdot \mathrm{lcm}(\kappa_+, |\kappa_-|). \tag{6}$$

This is indeed true if one takes the following interpretation of the frequencies (slight modification of Maggiore's conjecture): the straightforward extension of Maggiore's conjecture to the multi-horizon case is misleading, in fact only the equispaced families carry information about the quantum black hole mass transitions. (Every frequency belongs to one of the families.) Thus one has to first identify the equispaced families and then take the limit in the spacing in the imaginary part of the frequencies within each of the families. Such interpretation then fixes together with the formula (3) the parameter γ to be $\gamma = 1$. This means the area spectra of both of the horizons are given as $8\pi n$. Let us remind here, that the same analysis can be repeated for both S-dS and R-N-dS spacetimes: Assuming that all the horizons have the same equispaced area spectra, the single M parameter transitions lead to the surface gravities rational ratio condition and the QNM frequencies given by the formula fix the spectra of all the horizons to be $8\pi n$, (after one considers our generalization/modification of Maggiore's conjecture).

3 Conclusions

To summarize: We suppose that also in the multi-horizon case the modified Hod's conjecture provides information about the spacetime horizons spectra, only the way

[1] This type of spectrum with $\gamma = 1$ is the one originally suggested by Bekenstein for the black hole horizon area and represents the far most popular choice in the current literature. Let us also mention that by different, but not too different lines of thinking as we present, it was already speculated that both of the horizons in the R-N spacetime have the same area spectra of the Bekenstein form.

the information is encoded is more tricky than in the single horizon case. (This is hardly anything surprising as the quantization of more than one horizon might play a role in the game.) The QNM frequencies are consistent with each of the horizons being quantized with spectra given as $8\pi n$ (in Planck units). If these conclusions are accepted then still many open questions remain, for example, if similar interpretation could survive in case of charged black holes that occur after a collapse of matter. (In such case the black hole interior is very different to the extremely idealized R-N, R-N-dS cases. For example a weak mass-inflation singularity occurs at the inner horizon, but some results indicate that possibility of crossing the inner horizon might still be considered.)

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