Gravitomagnetism: From Einstein's 1912 Paper to the Satellites LAGEOS and Gravity Probe B

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Abstract The first concrete calculations of (linear) gravitomagnetic effects were performed by Einstein in 1912-1913. Einstein also directly and decisively contributed to the "famous" papers by Thirring (and Lense) from 1918. Generalizations to strong fields were performed not earlier than in 1966 by Brill and Cohen. Extensions to higher orders of the angular velocity ω by Pfister and Braun (1985-1989) led to a solution of the centrifugal force problem and to a quasiglobal principle of equivalence. The difficulties but also the recent successes to measure gravitomagnetic effects are reviewed, and cosmological and Machian aspects of gravitomagnetism are discussed.

1 Einstein's papers on gravitomagnetism from 1912 and 1913

Einstein's paper "Is there a gravitational action analogous to electromagnetic induction?" [1] from July 1912 (presumably his last work in Prague) is exceptional in many ways: It is published in a journal for forensic medicine (as a birthday present for his friend Heinrich Zangger), and it is very short (4 pages in the original setting, equivalent to less than 1.5 pages in today's Physical Review). It introduces audacious new concepts: the model of a spherical mass shell with mass M and radius R (which is useful until today in general relativity, because it is the optimal substitute for Newton's mass point, and because it allows to treat systems with matter by solving only the vacuum equations of general relativity), moreover a new gravitomagnetic "force", and the first calculation of a dragging effect: If the mass shell is linearly accelerated with Γ, Einstein calculates that a test mass m at the center of the shell is dragged with acceleration $\gamma = \frac{3}{2}(M/R)\Gamma$ (in units with $G = c = 1$).

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On the other hand, from today's perspective of general relativity, most details of the paper are wrong or inconsistent: The calculated mass increase of the test mass $m \to m' = m + mM/R$ is only a coordinate effect in general relativity [2]; in the calculated linear dragging acceleration γ the prefactor $\frac{3}{2}$ has to be substituted by $\frac{4}{3}$ [3], and, most importantly, a scalar relativistic gravity theory (which was the basis of Einstein's paper) can never produce a vectorial gravitomagnetic induction.

But the central new physical ideas of this paper (dragging and gravitomagnetism) kept command over Einstein when in 1913 (now in Zürich, with M. Grossman) he formulated the tensorial Entwurf theory. In the so-called Einstein-Besso manuscript [4] of June 1913 they calculated within this theory, besides the main topic of perihelion advance of Mercury, also a new value for the linear dragging acceleration ($\gamma = 2(M/R)\Gamma$), a Coriolis force inside a rotating spherical mass shell, and therefrom a rotational dragging of test masses (half the value in final general relativity), and a motion of the nodes of planets in the field of the rotating sun (1/4 of the value in general relativity). It is quite interesting which parts of this manuscript Einstein presented in his great and brilliant speech at the Naturforscher-Versammlung in Vienna in September 1913 [5], and which parts he omitted. When Einstein had finished general relativity in November 1915, he did not immediately come back to the questions of dragging and gravitomagnetism, because there were more urgent new problems (gravitational waves, cosmology, gravitational field energy, ...), and because he presumably imagined that the results on dragging and gravitomagnetism in general relativity would be similar to his results in the Entwurf theory.

2 The papers of Thirring (and Lense) on gravitomagnetism from 1918

It is well known that questions of dragging and gravitomagnetism in general relativity were first taken up in 1917-1918 by Hans Thirring (and J. Lense). Not so well known is that these papers owe nearly all their interesting and correct results to the direct interference of Einstein. Thirring had started his work in April 1917 (see [6] and [7]) with (partly wrong) calculations of centrifugal effects exerted by rotating mass shells and full bodies, and he did not realize that these effects, being of second order in the angular velocity ω, are ridiculously small for all laboratory and solar systems. In a letter of July 17 [8], Thirring informed Einstein about his work, together with some questions. Einstein's answer of August 2 [8] is short and polite, but admirably clear and concise. He stresses that much more important and realistic than centrifugal effects are Coriolis effects of first order of ω; he explains to Thirring the resulting dragging phenomena and the effects on the planets and moons in the solar system, and tells him that he has calculated all these effects (in the Entwurf theory), a fact which should have been known to Thirring from Einstein's speech [5] in Vienna in 1913. Only after this eye-opening lesson from Einstein is Thirring able to produce his two "famous" papers [9] and [10] of 1918. Still these papers have severe deficits: For the rotating mass shell Thirring calculates in the weak field approximation the dragging acceleration of test masses of velocity $\mathbf{v}$: $\mathbf{a} = 2d\,\mathbf{v} \times \omega$, with $d_{Th} = 4M/3R$, but only near the center of the shell (for $r \ll R$), and for the rotating full body he calculates the so-called Lense-Thirring effect $\mathbf{a} = 2\mathbf{v} \times \mathbf{H}$, with the gravitomagnetic dipole field $\mathbf{H} = \frac{2MR^2}{5r^3}[\omega - 3(\omega\mathbf{r})\mathbf{r}/r^2]$ only for $r \gg R$, which does not apply to the modern satellite experiments LAGEOS and Gravity Probe B. (See section 5 below.) The centrifugal results of order ω^2 in [9] contain many errors: an integration error observed by Laue and Pauli in 1920, the error (observed by Lanczos [11]) that Thirring modelled the mass shell as dust, and did therefore not correctly solve the Einstein equations, and the result of an axial component of his centrifugal "force", for which he gave a wrong physical explanation. The contributions of J. Lense to [10] are anyhow only of minor, technical character: The transformation of Thirring's results from Cartesian coordinates to the orbital elements used in astronomy, and their evaluation for some planets and moons of the solar system.

In my judgement a more original and valuable (but seldom quoted) paper by Thirring is his [12] where he as the first person (and correctly) formulates the analogies between electromagnetism and the Einstein equations in linear approximation, discusses the different signs and a factor 4 of the basic equations of gravitomagnetism in comparison to electromagnetism, and here he even mentions the preliminary discussion of gravitomagnetism by Einstein in his Vienna speech [5] of 1913. (For a modern and more extended treatment of gravitomagnetism see [13].)

3 Generalizations to strong fields and higher orders of ω. Solution of the centrifugal force problem

Considerable progress and extension of the work of Einstein and Thirring happened only in 1966 by the work of Brill and Cohen [14] who performed a first order rotational perturbation not of Minkowski spacetime but of the Schwarzschild solution, with the result for the dragging factor $d_{BC} = 4\alpha(2-\alpha)/((1+\alpha)(3-\alpha))$, with $\alpha = M/2R$, where R is the shell radius in isotropic coordinates. The important new physical result is that in the collapse limit $\alpha \to 1$ the factor d_{BC} attains the value 1: total dragging, and herewith a complete realization of the Machian postulate of relativity of rotation: in this limit the interior of the shell cuts itself off as a type of separate universe, and interior test particles are dragged along with the full angular velocity ω of the shell. As far as I know, Brill and Cohen were also the first to make clear that the interior Coriolis field applies to all $r < R$, and the exterior dipole field to all $r > R$. (The latter follows simply from symmetry arguments: a first order rotational perturbation of a spherical system produces quite generally a pure dipole field proportional to r^{-3}.)

An extension of this work to higher orders of ω, and in particular the problem of the notoriously wrong "centrifugal force" inside a rotating mass shell had to wait for another 19 years to be solved in [15]. The solution is based on two "new" observations which could and should have been made already in Thirring's time, but which were overlooked by all authors before 1985:

a) Any physically realistic rotating body will suffer a centrifugal deformation in orders ω^2 and higher, and cannot be expected to keep its spherical shape.
b) If we aim to realize inside the rotating mass shell quasi-Newtonian conditions with correct Coriolis and centrifugal forces — and no other forces! — , the interior of the mass shell obviously has to be a flat piece of spacetime. In the first order of ω this flatness is more or less trivial; however, in order ω^2 it is by no means trivial, and is indeed violated for Thirring's solution, due to the axial component of his "centrifugal force".

These observations lead to the mathematical question whether it is possible to connect a rotating flat metric through a mass shell (with, to begin with, unknown geometrical and material properties) to the non-flat but asymptotically flat exterior metric of a rotating body. In [15], [16] and [17] we could show that this problem has (for given M, R, and $\omega \ll 1/R$) a unique solution in every order ω^n, and that the resulting mass shell has non-spherical (surprisingly oblate) geometry, non-spherical mass distribution, and differential rotation. Only in the collapse limit $R \to M/2$ the shell is again spherical and rigidly rotating, as was already deduced by de la Cruz und Israel [18].

4 A quasi-global principle of equivalence

The success with this "matter-induced centrifugal force" guided me to the following hypothesis of a "quasi-global equivalence principle in general relativity" [15]. In short: "Every acceleration field can be understood as a gravitational field." In more detail: If some finite laboratory (a flat region of spacetime) is in arbitrary accelerated motion relative to the fixed stars, then all motions of free particles and all physical laws, measured from laboratory axes, are modified by inertial forces. It is argued that exactly the same modified motions and laws can be induced (at least for some time) at all places of a laboratory at rest relative to the fixed stars, by suitable and suitably accelerated masses outside the laboratory, e.g., in a mass shell. After formulating this hypothesis in 1985, I found that similar ideas arose already in the years 1912-1913 in discussions of Einstein with Ehrenfest [19] and Mie [5]. But at that time these people were quite sceptical about such a "macroequivalence". Today there are good arguments for the validity of the hypothesis at least for small accelerations because for small rotations (in [15]) and small linear accelerations (in [3]) the hypothesis has been explicitly proven, and because arbitrary accelerations can (at least in principle) be combined from linear and rotational accelerations.

5 Measuring gravitomagnetism

I should like to comment on the difficulties but also successes to measure the new "force" gravitomagnetism. For laboratories on earth and for satellites we have on one hand a factor $M_{\text{earth}}/R_{\text{earth}} \approx 10^{-9}$ for any deviations from Newtonian gravity. For rotational effects there comes another factor $\omega_{\text{earth}} R_{\text{earth}}/c \approx 10^{-6}$, therefore a factor 10^{-15} for any gravitomagnetic field, in comparison to Newtonian gravity. (Already Einstein in his letter to Thirring from 1917 stated that "the effects stay far below the measurement error".) Since there exist no gravitomagnetic materials in nature, there comes typically another factor $v/c \leq 10^{-5}$ from the velocity v of the rotating parts of the measuring device (except where these are photons or neutrinos). The resulting demand of a total precision of 10^{-20} can presumably not be fulfilled by any laboratory experiment in the foreseeable future, why I judge all pertaining recent proposals as questionable, even if they use Bose-Einstein condensates as in [20]. For neutron stars, pulsars, and black holes the above numbers are of course much more favourable. But in these astrophysical systems there exist many competing, partly unknown or poorly understood processes so that it is again questionable whether they lead to a clear measurement of gravitomagnetism [21].

In contrast, already soon after the start of the first earth satellites (in 1957) there appeared proposals (e.g., by V. Ginzburg and L. Schiff) to use these for tests of general relativity, because in space there is automatically high vacuum and low temperature, and because such tests can accumulate data over long time (years). In an admirable effort over 40 years (and with expenses of 700 million US$) the Stanford Gravity Probe B project (a satellite with $r/R \approx 1.10$) has finally confirmed the Lense-Thirring or rather Schiff effect (precession of a gyroscope axis) with 19% precision, much less than the originally expected precision of 1% ([22]). (The accompanying geodetic precession is not a gravitomagnetic effect, because the "gravitomagnetic invariant" ${}^*R \cdot R = \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\mu\nu} R^{\mu\nu}_{\gamma\delta}$ is zero for this effect.) A somewhat better (10%) confirmation of the Lense-Thirring effect was, however, performed already some years earlier by Ciufolini et al. [23] by a (in principle) much simpler satellite experiment: the careful measurement of the orbits of the passive satellites LAGEOS I and II (with $r/R \approx 1.92$) over 11 years, together with a precise measurement of the earth multipole moments $J_2, J_4, \ldots$ by the satellites CHAMP and GRACE. An ingenious proposal by Ciufolini [24] to start LAGEOS II with orbital elements "complementary" to LAGEOS I, and hereby cancelling the multipole contributions, was unfortunately never realized. But the newly launched satellite LARES gives hope to confirm a gravitomagnetic effect soon with 1% precision.

If gravitational waves can be analyzed in detail in the future, this will also be an indirect test for gravitomagnetism, because, similar to electromagnetism, gravitational waves have in equal parts gravitoelectric and gravitomagnetic contributions.

6 Cosmological remarks

Although the dragging results of Einstein, Thirring, Brill-Cohen et al. with their asymptotically flat solutions do not really meet the Machian demand for a cosmological origin of inertia, it was proven by my PhD-student C. Klein [25], and by Bičák, Lynden-Bell, Katz [26], and by C. Schmid [27] that rotational perturbations of standard FRW cosmologies provide similar dragging results. Concerning the observational confirmation of the (non-causal!) determination of the local inertial frames by the cosmos as a whole, I should like to quote from the MTW-book [28]: "Consider a bit of solid ground near the geographic pole, and a support erected there, and from it hanging a pendulum. Though the sky is cloudy, the observer watches the track of the Foucault pendulum as it slowly turns through 360^o. Then the sky clears and, **miracle of miracles,** the pendulum is found to be swinging all the time on an arc fixed relative to the far-away stars." The presently best measurement of this "non-rotation" (smaller than 10^{-9} of the earth angular velocity) comes from the terrestrial reference system realized by VLBI and GPS [29].

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