Canonical Gravity, Non-Inertial Frames, Relativistic Metrology and Dark Matter

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Abstract Clock synchronization leads to the definition of instantaneous 3-spaces (to be used as Cauchy surfaces) in non-inertial frames, the only ones allowed by the equivalence principle. ADM canonical tetrad gravity in asymptotically Minkowskian space-times can be described in this framework. This allows to find the York canonical basis in which the inertial (gauge) and tidal (physical) degrees of freedom of the gravitational field can be identified. A Post-Minkowskian linearization with respect to the asymptotic Minkowski metric (asymptotic background) allows to solve the Dirac constraints in non-harmonic 3-orthogonal gauges and to find non-harmonic TT gravitational waves. The inertial gauge variable York time (the trace of the extrinsic curvature of the 3-space) describes the general relativistic freedom in clock synchronization. After a digression on the gauge problem in general relativity and its connection with relativistic metrology, it is shown that dark matter, whose experimental signatures are the rotation curves and the mass of galaxies, may be described (at least partially) as an inertial relativistic effect (absent in Newtonian gravity) connected with the York time, namely with the non-Euclidean nature of 3-spaces as 3-sub-manifolds of space-time.

While in special relativity (SR) the use of non-inertial frames is optional, in general relativity (GR) only these are allowed by the equivalence principle, forbidding the existence of global inertial frames. In both cases the Lorentz signature of the space-time implies that there is no notion of instantaneous 3-space: the only intrinsic structure is the conformal one, i.e. the light-cone as the locus of incoming and outgoing radiation. *A convention on the synchronization of clocks is needed to define an instantaneous 3-space*, where one has to give the Cauchy data for the relevant wave equations. For instance the *1-way velocity of light* from one observer A to an observer B has a meaning only after choosing such a convention. In SR Einstein convention for the synchronization of clocks in Minkowski space-time uses the 2-

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way (or round trip) velocity of light to identify the Euclidean 3-spaces of the inertial frames centered on an inertial observer A by means of only one clock. It is this velocity which is isotropic and constant in SR and replaces the standard of length in relativistic metrology [1]. Only in the inertial frames of SR the 1-way and the 2-way velocities coincide.

Therefore in Ref. [2, 3, 4] a general theory of global non-inertial frames in Minkowski space-time was developed by using the 3+1 point of view in which, besides the world-line of a time-like observer, one also gives a global, nice foliation of the space-time with instantaneous 3-spaces. In this way one avoids the coordinate singularities of the 1+3 description (both those of Fermi coordinates and of the rotating disk). The time-like observer carries a standard atomic clock and τ is an arbitrary monotonically increasing function of the proper time of this clock. The space-like instantaneous 3-spaces Σ_τ are the mathematical idealization of a protocol for clock synchronization: all the clocks in the points of Σ_τ show the same time of the atomic clock of the observer. On each 3-space Σ_τ one chooses curvilinear 3-coordinates σ^r having the observer as origin. The Lorentz-scalar and observer-dependent coordinates $\sigma^A = (\tau, \sigma^r)$ are named *radar 4-coordinates*. The coordinate transformation $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$ to the Cartesian coordinates x^μ defines the embedding $z^\mu(\tau, \sigma^r)$ of the 3-spaces Σ_τ into Minkowski space-time. The induced 4-metric on Σ_τ is the following functional of the embedding ${}^4g_{AB}(\tau, \sigma^r) = [z^\mu_A\, \eta_{\mu\nu}\, z^\nu_B](\tau, \sigma^r)$, where $z^\mu_A = \partial z^\mu / \partial \sigma^A$.

The 3+1 point of view has allowed to get the description of arbitrary isolated systems (particles, strings, fluids, fields) admitting a Lagrangian formulation in arbitrary non-inertial frames by means of *parametrized Minkowski theories* [2, 3, 4, 5, 6]. In them the Lagrangian is coupled to an external gravitational field and then the gravitational 4-metric is replaced with the 4-metric ${}^3g_{AB}(\tau, \sigma^r)$ induced by an admissible 3+1 splitting of Minkowski space-time. The new Lagrangian, a function of the matter and of the embedding, is invariant under frame-preserving diffeomorphisms and this type of general covariance implies that the embeddings are *gauge variables*, so that the transition among non-inertial frames is described as a *gauge transformation*: only the appearances change, not the physics. The metric ${}^3g_{AB}(\tau, \sigma^r)$ and the extrinsic curvature tensor ${}^3K_{rs}(\tau, \sigma^u)$ play the role of inertial potentials.

This framework allows us to define the *inertial and non-inertial rest frames* of the isolated systems, and to develop the rest-frame instant form of the dynamics and to build the explicit form of the Lorentz boosts for interacting systems. While the inertial rest frames have their Euclidean 3-spaces defined as space-like 3-manifolds of Minkowski space-time orthogonal to the conserved 4-momentum of the isolated system, the non-inertial rest frames are admissible non-inertial frames whose 3-spaces tend to those of some inertial rest frame at spatial infinity, where the 3-space becomes orthogonal to the conserved 4-momentum. This setting allows to study the problem of the relativistic center of mass with the associated external and internal (i.e. inside the 3-space) realizations of the Poincaré algebra in a way compatible with relativistic bound states [7, 8, 9, 10, 11], and to get a new Wigner-covariant formulation of relativistic quantum mechanics [12], with a solution of all the known problems introduced by SR.

This metrology-oriented solution of the problem of clock synchronization used in SR can be extended to GR, if Einstein space-times are restricted to the class of globally hyperbolic, topologically trivial, asymptotically Minkowskian space-times without super-translations and without Killing symmetries, which include the Christodoulou-Klainerman space-times [13, 14]. In these space-times one can define global non-inertial frames by using the same admissible 3+1 splittings, centered on a time-like observer, to define the 3-spaces Σ_τ and the observer-dependent radar 4-coordinates $\sigma^A = (\tau; \sigma^r)$ employed in SR. This will allow to separate the *inertial* (gauge) degrees of freedom of the gravitational field (playing the role of inertial potentials) from the dynamical *tidal* ones at the Hamiltonian level.

In GR the gradients $z^\mu_A(\tau, \sigma^r)$ of the embeddings $x^\mu = z^\mu(\tau, \sigma^r)$, defining the admissible 3+1 splittings of space-time, give the transition coefficients from radar to world 4-coordinates.

The components ${}^4g_{AB}(\tau, \sigma^r) = z^\mu_A(\tau, \sigma^r)\, z^\nu_B(\tau, \sigma^r)\, {}^4g_{\mu\nu}(z(\tau, \sigma^r))$ of the 4-metric will be the dynamical fields in the ADM action [15], written in the basis of radar 4-coordinates. Like in SR the 4-vectors $z^\mu_u(\tau, \sigma^r)$, tangent to the 3-spaces Σ_τ, are used to define the unit normal $l^\mu(\tau, \sigma^r) = z^\mu_A(\tau, \sigma^r)\, l^A(\tau, \sigma^r)$ to Σ_τ, while the 4-vector $z^\mu_\tau(\tau, \sigma^r)$ has the lapse function as component along the unit normal and the shift functions as components along the tangent vectors.

While in SR time and 3-space are absolute notions, in GR the space-time is a dynamical object [16, 17, 18]. Each solution (i.e. an Einstein 4-geometry) of Einstein's equations (or of the associated ADM Hamilton equations) dynamically selects a preferred 3+1 splitting of the space-time, namely in GR the instantaneous 3-spaces are dynamically determined in the chosen world coordinate system, modulo the choice of the 3-coordinates into the 3-space and modulo the trace of the extrinsic curvature of the 3-space as a space-like sub-manifold of the space-time [19]. In GR the gravitational field is described by the ten dynamical fields ${}^4g_{\mu\nu}(x)$, which also determine the *chrono-geometrical structure of space-time* through the line element $ds^2 = {}^4g_{\mu\nu}\, dx^\mu\, dx^\nu$. Therefore the 4-metric teaches relativistic causality to the other fields: it says to massless particles like photons and gluons which are the allowed world-lines in each point of space-time.

As shown in the first paper of Ref. [20, 21], in the chosen class of space-times the 4-metric ${}^4g_{\mu\nu}(x)$ tends in a suitable way to the flat Minkowski 4-metric ${}^4\eta_{\mu\nu}$ at spatial infinity (to be used as an *asymptotic background* at spatial infinity in the linearization of the theory), where there are asymptotic inertial observers whose spatial axes may be identified by means of the fixed stars of star catalogues (the fixed stars can be considered as an empirical definition of spatial infinity of the observable universe). In absence of super-translations the asymptotic symmetries reduce to the asymptotic ADM Poincaré group. The ten *strong* asymptotic ADM Poincaré generators P^A_{ADM}, J^{AB}_{ADM} (they are fluxes through a 2-surface at spatial infinity) are well defined functionals of the 4-metric fixed by the boundary conditions at spatial infinity. Moreover in that paper it is also shown that the boundary conditions on the 4-metric required by the absence of super-translations imply that the only admissible 3+1 splittings of space-time (i.e. the allowed global non-inertial frames) are the *non-inertial rest frames*: their 3-spaces are asymptotically orthogonal to the weak ADM 4-momentum. Therefore one gets $\hat{P}^r_{ADM} \approx 0$ as the rest-frame condition of the 3-universe with a mass and a rest spin fixed by the boundary conditions.

Finally, in the limit of vanishing Newton's constant ($G = 0$) the asymptotic ADM Poincaré generators become the generators of the special relativistic Poincaré group describing the matter present in the space-time, allowing the inclusion into GR of the classical version of the standard model of particle physics, whose properties are all connected with the representations of this group in the inertial frames of Minkowski space-time.

To define the canonical formalism the Einstein-Hilbert action for metric gravity (depending on the second derivative of the metric) must be replaced with the ADM action (the two actions differ by a surface term at spatial infinity). As shown in the first paper of Refs. [20, 21], the Legendre transform and the definition of a consistent canonical Hamiltonian require the introduction of the DeWitt surface term at spatial infinity: the final canonical Hamiltonian turns out to be the *strong* ADM energy (a flux through a 2-surface at spatial infinity), which is equal to the *weak* ADM energy (expressed as a volume integral over the 3-space) plus constraints. Therefore there is not a frozen picture like in the "spatially compact space-times without boundaries" used in loop quantum gravity (where the canonical Hamiltonian vanishes), but an evolution generated by a Dirac Hamiltonian equal to the weak ADM energy plus a linear combination of the first class constraints. Also the other strong ADM Poincaré generators are replaced by their weakly equivalent weak form $\hat{P}^A_{ADM}$, $\hat{J}^{AB}_{ADM}$.

To take into account the fermion fields present in the standard particle model one must extend ADM gravity to ADM tetrad gravity. Since our class of space-times admits orthonormal tetrad and a spinor structure [22], the extension can be done by simply replacing the 4-metric in the ADM action with its expression in terms of cotetrad fields $E^{(\alpha)}_A(\tau,\sigma^r)$,

$$ {}^4g_{AB}(\tau,\sigma^r) = E^{(\alpha)}_A(\tau,\sigma^r)\, {}^4\eta_{(\alpha)(\beta)}\, E^{(\beta)}_B(\tau,\sigma^r) $$

((α) are flat indices and ${}^4\eta_{(\alpha)(\beta)}$ the flat metric; by convention a sum on repeated indices is assumed). The cotetrad fields $E^{(\alpha)}_A$, considered as the basic 16 configurational variables in the ADM action, are the inverse of the tetrad fields $E^A_{(\alpha)}$, which are connected to the world tetrad fields by $E^\mu_{(\alpha)}(x) = z^\mu_A(\tau,\sigma^r)\, E^A_{(\alpha)}(z(\tau,\sigma^r))$. The cotetrads $E^{(\alpha)}_A(\tau,\sigma^r)$ are connected to cotetrads ${}^4\overset{\circ}{E}{}^{(\alpha)}_A(\tau,\sigma^r)$ adapted to the 3+1 splitting of space-time, namely such that the inverse adapted time-like tetrad ${}^4\overset{\circ}{E}{}^A_{(o)}(\tau,\sigma^r)$ is the unit normal to the 3-space Σ_τ, by a standard Wigner boosts for time-like Poincaré orbits with parameters $\varphi_{(a)}(\tau,\sigma^r)$, $a = 1,2,3$.

This leads to an interpretation of gravity based on a congruence of time-like observers endowed with orthonormal tetrads: at each point of space-time the time-like axis is the unit 4-velocity of the observer, while the spatial axes are a (gauge) convention for observer's gyroscopes. This framework was developed in the second and third paper of Refs. [20, 21].

Even if the action of ADM tetrad gravity depends upon 16 fields, the counting of the physical degrees of freedom of the gravitational field does not change, because this action is invariant not only under the group of 4-diffeomorphisms but also under the O(3,1) gauge group of the Newman-Penrose approach [23] (the extra gauge freedom acting on the tetrads in the tangent space of each point of space-time).

After having introduced the kinematical framework for the description of non-inertial frames in GR, we must study the dynamical aspects of the gravitational field to understand which variables are dynamically determined and which are the inertial effects hidden in the general covariance of the theory. Since at the Lagrangian level it is not possible to identify which components of the 4-metric tensor are connected with the gauge freedom in the choice of the 4-coordinates and which ones describe the dynamical degrees of freedom of the gravitational field, one must restrict oneself to the quoted class of globally hyperbolic, asymptotically Minkowskian space-times allowing a Hamiltonian description of ADM gravity. In canonical ADM gravity one can use Dirac theory of constraints to describe the Hamiltonian gauge group, whose generators are the first-class constraints of the model. The basic tool of this approach is the possibility to find so-called Shanmugadhasan canonical transformations [24, 25], which identify special canonical bases adapted to the first-class constraints (and also to the second-class ones when present). In these special canonical bases the vanishing of certain momenta (or of certain configurational coordinates) corresponds to the vanishing of well defined Abelianized combinations of the first-class constraints (Abelianized because the new constraints have exactly zero Poisson brackets even if the original constraints were not in strong involution). As a consequence, the variables conjugate to these Abelianized constraints are *inertial Hamiltonian gauge variables describing the Hamiltonian gauge freedom*.

Therefore, starting from the ADM action for tetrad gravity one defines the Hamiltonian formalism in a phase space containing 16 configurational field variables and 16 conjugate moments. One identifies the 14 first-class constraints of the system. The existence of these 14 first-class constraints implies that 14 components of the tetrads (or of the conjugate momenta) are Hamiltonian gauge variables describing the *inertial* aspects of the gravitational field (6 of these inertial variables describe the extra gauge freedom in the choice of the tetrads and in their transport along world-lines). Therefore there are only 2+2 degrees of freedom for the description of the *tidal* dynamical aspects of the gravitational field (the two polarizations of gravitational waves in the linearized theory). The asymptotic ADM Poincaré generators can be evaluated explicitly. Till now the type of matter studied in this framework [26, 27, 28] consists of the electromagnetic field and of N charged scalar particles, whose signs of the energy and electric charges are Grassmann-valued to regularize both the gravitational and electromagnetic self-energies (it is both a ultraviolet and an infrared regularization).

If one would be able to include all the constraints in the Shanmugadhasan canonical basis, the 2+2 tidal variables would be the *Dirac observables* of the gravitational field, invariant under the Hamiltonian gauge transformations. However such Dirac observables are not known: one only has statements about their existence. Moreover, in general they are not 4-scalar observables. The problem of the connection between the 4-diffeomorphism group and the Hamiltonian gauge group was studied in Ref. [29, 30] by means of the inverse Legendre transformation and of the notion of dynamical symmetry. The conclusion is that on the space of solutions of Einstein equations there is an overlap of the two types of observables: there should exist special Shanmugadhasan canonical bases in which the 2+2 Dirac observables become 4-scalars when restricted to the space of solutions of the Einstein equations (i.e. on-shell). In any case the identification of the inertial gauge components of the 4-metric is what is needed to make a fixation of 4-coordinates as required by relativistic metrology.

The best which can be done till now is the explicit identification of a Shanmugadhasan canonical transformation [19] (implementing the so-called York map and diagonalizing the York-Lichnerowicz approach) to a so-called York canonical basis adapted to 10 of the 14 first-class constraints. Only the super-Hamiltonian and super-momentum constraints, whose general solution is not known, are not included in the basis, but it is clarified which variables are to be determined by their solution, namely the 3-volume element (the determinant of the 3-metric) of the 3-space Σ_τ and the three momenta conjugated to the 3-coordinates on Σ_τ. The 14 inertial gauge variables turn out to be: a) the six configurational variables $\varphi_{(a)}$ and $\alpha_{(a)}$ of the tetrads describing their O(3,1) gauge freedom; b) the lapse and shift functions; c) the 3-coordinates on the 3-space (their fixation implies the determination of the shift functions); d) the York time ${}^3K$, i.e. the trace of the extrinsic curvature of the 3-spaces as 3-manifolds embedded into the space-time (its fixation implies the determination of the lapse function). It is the only gauge variable which is a momentum in the York canonical basis (instead in Yang-Mills theory all the gauge variables are configurational): this is due to the Lorentz signature of space-time, because the York time and three other inertial gauge variables can be used as 4-coordinates of the space-time. In this way an identification of the inertial gauge variables to be fixed to get a 4-coordinate system in relativistic metrology was found. While in SR all the components of the tetrads and their conjugate momenta are inertial gauge variables, in GR the two eigenvalues of the 3-metric with determinant one and their conjugate momenta describe the physical tidal degrees of freedom of the gravitational field. In the first paper of Ref. [26, 27, 28] there is the expression of the Hamilton equations for all the variables of the York canonical basis.

An important remark is that in the framework of the York canonical basis the natural family of gauges is not the harmonic one, but the family of 3-orthogonal Schwinger time gauges in which the 3-metric in the 3-spaces is diagonal [26, 27, 28].

In conclusion, while the gauge group of the Lagrangian formulation of Einstein GR, the diffeomorphism group, implies that the 4-coordinates of the space-time are *gauge variables*, the Hamiltonian gauge group replaces them with the *inertial gauge variables* York time and 3-coordinates on the instantaneous 3-space Σ_τ. In both cases one would like to re-express physical properties in terms either of 4-scalars or of Dirac observables becoming 4-scalars on-shell. However, on one side it is not yet known how to implement this program and on the other side this is not the praxis of experimental physics.

Inside the Solar System the experimental localization of macroscopic classical objects is unavoidably done by choosing some *convention* for the local 4-coordinates of space-time. Atomic physicists, NASA engineers and astronomers have chosen a series of reference frames and standards of time and length suitable for the existing technology [1, 31, 32, 33]. These conventions determine certain Post-Minkowskian (PM) 4-coordinate systems (in harmonic gauges) of an asymptotically Minkowskian space-time, in which the instantaneous 3-spaces are not strictly Euclidean. Then these reference frames are seen as a local approximation of a celestial reference frame (ICRS), where however the space-time has become a cosmological Friedman-Robertson-Walker (FRW) one, which is only conformally asymptotically Minkowskian at spatial infinity and therefore does not admit a Hamiltonian description. A search of a consistent patching of the 4-coordinates from inside the Solar System to the rest of the universe will start when the data from the future GAIA mission [34] for the cartography of the Milky Way will be available. This will allow a PM definition of a Galactic Reference System containing at least our galaxy. Let us remark that notwithstanding the FRW instantaneous 3-spaces are not strictly Euclidean, all the books on galaxy dynamics describe the galaxies by means of the Kepler theory in Galilei space-time.

Both in SR and GR an admissible 3+1 splitting of space-time has two associated congruences of time-like observers [2, 3, 4], geometrically defined and not to be confused with the congruence of the world-lines of fluid elements, when relativistic fluids are added as matter in GR [35]. One of the two congruences, with zero vorticity, is the congruence of the Eulerian observers, whose 4-velocity field is the field of unit normals to the 3-spaces. This congruence allows us to re-express the non-vanishing momenta of the York canonical basis in terms of the expansion ($\theta = -{}^3K$) and of the shear of the Eulerian observers. This allows us to compare the Hamilton equations of ADM canonical gravity with the usual first-order non-Hamiltonian ADM equations deducible from Einstein equations given a 3+1 splitting of space-time but without using the Hamiltonian formalism. As a consequence, one can extend our Hamiltonian identification of the inertial and tidal variables of the gravitational field to the Lagrangian framework and use it in the cosmological (conformally asymptotically flat) space-times: in them it is not possible to formulate the Hamiltonian formalism but the standard ADM equations are well defined. The time inertial gauge variable needed for relativistic metrology is now the expansion of the Eulerian observers of the given 3+1 splitting of the globally hyperbolic cosmological space-time. It is this inertial gauge variable which has to be fixed in this way to reproduce experimental astronomical data and their astrophysical interpretation.

In conclusion we now have a framework for non-inertial frames in GR and an identification of the inertial gauge variables in asymptotically Minkowskian and also in cosmological space-times. See the third paper of Refs. [26, 27, 28] for the possibility that the three main signatures of dark matter (rotation curves of galaxies; mass of galaxy clusters from the virial theorem and weak gravitational lensing; see the review in Ref. [36, 37]) can be explained as only a relativistic inertial effect induced by the inertial gauge variable ${}^3K$ (the York time): a suitable choice of the 3-space in the celestial reference frame could simulate the effects explained with dark matter. Since in the PM Hamiltonian linearization of canonical tetrad gravity [26, 27, 28] the lapse function is $n = n_1 + \partial_\tau {}^3K + ..$ with n_1 describing the Newtonian potential in the non-relativistic limit, one can identify $\partial_\tau {}^3K$ with the Yukawa-like potential used in $f(R)$ gravity to simulate dark matter [38].

See Ref. [39] for an extended and complete review of the approach. In this paper it is also shown (at a preliminary level) that the York time is connected also with dark energy in inhomogeneous cosmological space-times [40].

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