Phase Structure of Five Dimensional Black Di-ring

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Abstract We investigate the phase structure of black di-ring in five-dimensional asymptotically flat vacuum gravity. We numerically plot the points of black di-rings in the phase diagram to study the region covered by black di-ring. The distribution of black di-ring shows that the area of black di-ring is always less than the maximum value of black ring. The plot indicates that there are black di-ring configurations whose area parameters are arbitrarily close to zero.

1 Introduction

In five dimensions, in addition to the solutions with a single horizon, there exist solutions with disconnected event horizons. Black Saturn which is a spherical black hole surrounded by a black ring was constructed by the inverse scattering method [1]. It was shown that the black rings can be superposed concentrically by using the Bäcklund transformation [2]. This black di-ring solution also can be constructed by the inverse scattering method [3].

The existence of multi-black hole configurations implies continuous non-uniqueness of five-dimensional black holes. The phase diagram of the black Saturn was investigated in [4, 1]. The plot of random sets of points in the phase diagram showed that the black Saturn covers the wide region of the phase diagram. The phases of black Saturn were investigated by the method based on the thin and long ring approximation: the black Saturn can be modeled as a simple superposition of a Myers-Perry black hole and a very thin black ring [1]. It was argued that the configurations that approach maximal entropy for fixed mass and angular momentum are black Saturns with a nearly static black hole and a very thin black ring.

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The black di-ring also indicates “an infinite non-uniqueness” [2]. It was confirmed that there are infinite number of black di-rings for the same mass and angular momentum. The phase structure of a black di-ring in thermodynamic equilibrium was investigated in [5, 6]. Distributions of black di-rings in the phase diagram have not been fully investigated. When we approximate the black di-ring as a simple superposition of two concentric black rings, we can roughly estimate the region covered by the black di-ring in the phase diagram. The maximum of the area would be smaller than the one of black Saturn for the same mass and angular momentum. Because of the strong non-linearity, however, we need rigorous analysis for the distributions of black di-ring in the phase diagram for a decisive conclusion.

2 Rod structure of black di-ring

The rod structure analysis is a very useful tool to understand a higher dimensional black hole solution. The rod structure of black di-ring is composed by two semi-infinite rods and four finite rods. Two of four finite rods are timelike and the other two finite and two semi-infinite rods are spacelike. The finite timelike rods have direction vectors $(1, \Omega_i, 0)$ if the solution has angular momentum along S^1 direction. The rod structure of black di-ring rotating along S^1 direction is described in Fig. 2 (left). This solution was constructed from the seed solution, whose rod structure is described in Fig. 2 (right), by the inverse scattering method [3]. The seed solution has six finite rods. We define six parameters by using the lengths of these finite rods as in Fig. 2. Physical variables of black di-ring are expressed by using these six parameters.

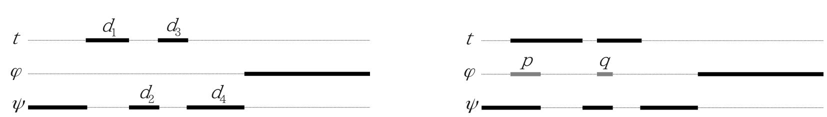

Fig. 1 *Left:* Rod structure of black di-ring. *Right:* Rod structure of seed solution of black di-ring with S^1 rotation.

3 Phase structure of black di-ring

The physical variables of black di-ring are calculated from the exact expressions of the solution. Following [1], we normalize the ADM angular momentum and the area of horizons as

$$j^2 = \frac{27\pi}{32G}\frac{J^2}{M^3}, \quad a_h = \frac{3}{16}\sqrt{\frac{3}{\pi}}\frac{A_h}{(GM)^{3/2}}, \tag{1}$$

to compare the physical properties of black objects with the same ADM mass.

We fix the scaling freedom by $d_1 + d_2 + d_3 + d_4 = 1$. The balance conditions impose two constraints on the parameters. As a result, the balanced black di-ring has three dimensionless parameters. In the analysis, we choose d_2, d_3 and d_4 as the three parameters for the balanced black di-ring. The parameter d_1 is determined by the scaling. The parameters p and q are determined by solving the balance conditions.

To investigate the region of the phase diagram covered by black di-ring, we plot the point (j^2, a_h) corresponding to the sets of parameters (d_2, d_3, d_4). The result is shown in Fig. 2 (left). Figure 2 (right) is a similar plot for black Saturns.

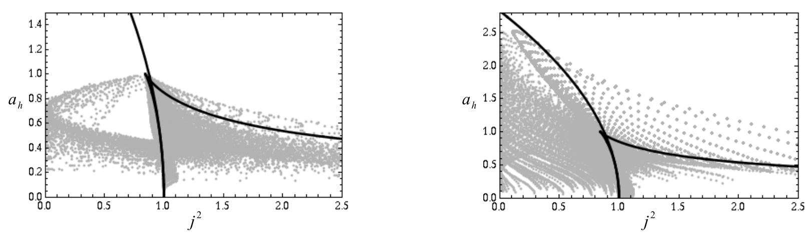

Fig. 2 *Left:* Distribution of black di-rings in the phase diagram. The black bold curves are the phases of the Myers-Perry black hole and the black ring. *Right:* Distribution of black Saturns in the phase diagram. The black bold curves are the phases of the Myers-Perry black hole and the black ring.

The total area of black di-ring can not become larger than the maximum of black ring $a_h = 1$. There are black di-ring configurations with total area a_h greater than the black ring with the same j^2. It can be confirmed that the black di-ring solution with $j = 0$ is possible while maintaining balance similar to black Saturn. In the plot of Fig. 2 (left) the low entropy black di-ring $a_h \lesssim 0.2$ is scarcely distributed except around $j^2 = 1$. When $d_1 = d_3 = 0$ the area of black di-ring becomes exactly zero. If we simply set $d_1 = d_3 = 0$, it can be easily shown that the balance condition for spacelike rod d_4 is violated. Therefore we have to choose parameters such that p and q become very small in addition to d_1 and d_3 for the small area black di-ring.

The low entropy black di-ring would be constructed by two different configurations. One is a double thin ring and the other is a combination of a nearly extremal fat ring with a large thin ring. The double thin ring configuration will be constructed by choosing the rod parameters as $d_1 \ll d_2$ and $d_3 \ll d_4$. The corresponding plot of the phase diagram becomes like Fig. 3 (left). The second configuration will be constructed by $d_1, d_3, d_4 \ll d_2$ and $d_3 \gg d_4$. The corresponding plot is given in Fig 3 (right). Both plots show that the black di-rings can exist in the region $0 < a < 0.2$ of the phase diagram.

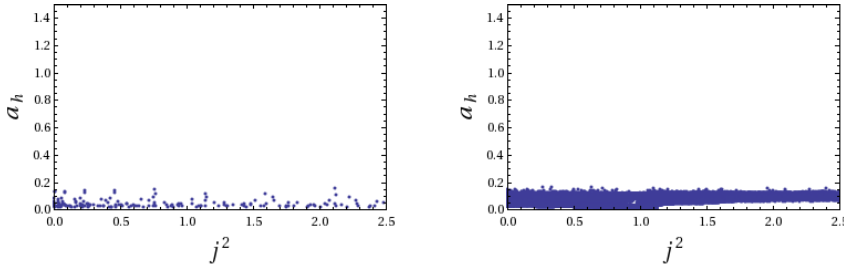

Fig. 3 *Left:* Plots for parameters which satisfy $d_1 \ll d_2$ and $d_3 \ll d_4$ *Right:* Plots for parameters which satisfy $d_1, d_3, d_4 \ll d_2$ and $d_3 \gg d_4$

4 Summary

We analyzed the phase structure of the black di-ring. The distribution of black di-rings in the phase diagram shows infinite non-uniqueness of the black di-ring. The configurations of black di-ring span the open strip $0 < a_h < 1$ and $j^2 > 0$.

Acknowledgements

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