Coupling Dimers to CDT to Obtain Higher Order Multicritical Behavior

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Abstract This contribution reviews some recent results on dimers coupled to CDT. A bijective mapping between dimers and tree-like graphs allows for a simple way to introduce dimers to CDT. This can be generalized further to obtain different multicritcal points.

1 Introduction

Causal Dynamical Triangulations (CDT) is a proposed theory of quantum gravity. In CDT the path integral for gravity is regularized through simplices as in dynamical triangulation. CDT introduces a preferred time slicing to provide for a well-defined Wick rotation. This preferred time slicing leads to a better behaved continuum theory [1] (see [2] for a review).

For matrix models it is well-known that random lattices can be coupled to matter, like dimers or the Ising model, to find quantum gravity coupled to conformal field theories [3, 4]. It is then an interesting prospect to try and couple matter to the random lattices of CDT.

In this article we review the results obtained in [5] and present a simple extension of the model which allows for higher order multicritical points.[1]

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[1] Quite similar results have been obtained simultaneously in [6, 7]

2 Coupling CDT to Dimers

Durhuus et al. [8] proved that there is a bijective mapping between rooted tree graphs and CDT (see Figure 1). This bijection makes it possible to determine the critical exponents of CDT using recursive equations as in [9]. It also makes it possible to consider the easier problem of coupling dimers to a rooted tree graph instead of directly placing them on the CDT. The simple rule of placing any number of hard dimers on the tree will lead to a partition function which allows for new multicritical behavior[5].

Hard dimers are a type of matter with local interactions. One can imagine a dimer like a fixed rod that can be added to the lattice on any link whose neighboring links are not taken up with dimers. This is illustrated in Figure 1 on the right.

The partition function for CDT with dimers reads

$$Z(\mu,\xi) = \sum_{BP} e^{-\mu} \sum_{\mathrm{HD(BP)}} \xi^{|\mathrm{HD(BP)}|}, \tag{1}$$

where BP is the set of all tree-like graphs, $\mathrm{HD(BP)}$ the set of all dimer configurations on that graph and $\mathrm{HD(BP)}$ the number of dimers in a given configuration. This partition function can be solved using recursive equations which arise for the tree like graphs and are discussed in detail in [9]. The recursion depicted in Figure 2

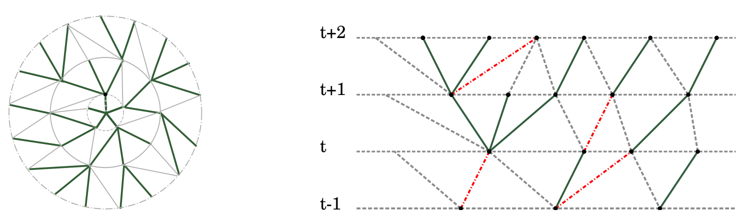

Fig. 1 On the left is an embedding of a simple CDT. The green marked lines on it are those that are also part of the graph. These suffice to characterize the entire CDT. If we only had the green lines we could get back to the full CDT by just reintroducing the space-like links and the leftmost link at every vertex. The figure on the right are three time slices of a CDT. The tree graph is marked green. The red markings indicate dimers, which can be placed on the tree so as to be non touching.

leads to the equations

$$Z = e^{-\mu}\left(\frac{1}{1-Z} + W\frac{1}{(1-Z)^2}\right), \qquad W = e^{-\mu}\xi\left(\frac{1}{1-Z}\right), \tag{2}$$

where Z is the partition function for a tree with a normal link at the root and W is the partition function for a tree rooted in a dimer. At a n-multicritical point the first $n-1$ derivatives of the coupling μ by the partition function Z are zero

$$\left.\frac{\partial\mu}{\partial Z}\right|_{Z_c} = \cdots = \left.\frac{\partial^{n-1}\mu}{\partial Z^{n-1}}\right|_{Z_c} = 0\,. \tag{3}$$

We can then solve equations (2) to find the third multicritical point at $Z_c = \frac{5}{8}$, $\xi_c = -\frac{1}{12}$ and $e^{\mu_c} = \frac{32}{9}$. The critical exponents at this point are $\gamma = \frac{1}{3}$, $d_H = \frac{3}{2}$ and $\sigma = \frac{1}{2}$. In pure CDT, not coupled to dimers, one finds $\gamma = \frac{1}{2}$, $d_H = 2$ and σ is not defined, so it is clear that CDT coupled to dimers lies in a different universality class than pure CDT. Therefore it represents an interacting system of matter and gravity. However the negative weight ξ does make the physical interpretation of the results less clear [10]. It is easy to generalize this model to higher order multicritcal points. To do

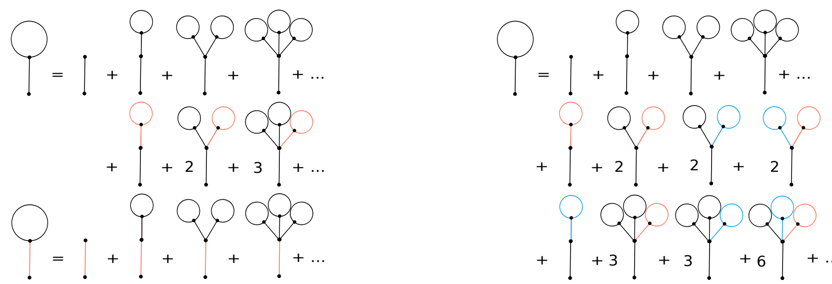

Fig. 2 These figures are pictorial representations of equations (2),(5). Dimers are depicted as red (blue) links.The upper left figure is equivalent to the first equation in (2) while the lower left is the second equation there. The right figure corresponds to equation (5).

so one introduces different types of dimers, denoted as type a with weight ξ and type b with weight ζ. The rule is then that a vertex with an incoming dimer can not spawn any type of dimer, while a vertex with an incoming empty link can spawn at most one dimer of each color. This is illustrated in Figure 2. This model leads to the partition function

$$Z(\mu,\xi,\zeta) = \sum_{\mathrm{BP}} e^{-\mu} \sum_{\mathrm{HD(BP(a,b))}} \xi^{|a|}\zeta^{|b|}, \tag{4}$$

where $\mathrm{HD(BP(a,b))}$ denotes the set of configurations of hard dimers of type a and b and $|a|$ ($|b|$) is the number of dimers of type a (b) in the configuration. It can again be solved using recursive equations for the tree graphs

$$Z = e^{-\mu}\left(\frac{1}{1-Z} + \frac{1}{(1-Z)^2}(W+V) + \frac{WV}{(1-Z)^3}\right), \tag{5}$$

where W denotes the partition function starting in a dimer of type a and V for dimers of type b. For W and V we obtain equations like (2).

This model has one multicritical point of fourth order at

$$(\xi_c,\zeta_c) = \left(\frac{1}{90}\left(5 \mp i\sqrt{35}\right), \frac{1}{90}\left(-5 \pm i\sqrt{35}\right)\right) \quad \text{and} \quad e^{-\mu_c} = \frac{256}{75}.$$

The critical exponents are $\gamma = \frac{1}{4}$ and $d_H = \frac{4}{3}$. It is possible to extend this model to any further multicritical point by introducing additional colors of dimers.

3 Summary

Introducing dimer-like matter to CDT leads to new critical behavior. This means that there is a coupling between the quantum gravity of CDT and the matter of the dimers. Through the introduction of different types of dimers it is possible to obtain multicritical points of any order.

Acknowledgments

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