

Loop Quantum Gravity and the The Planck Regime of Cosmology

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Abstract The very early universe provides the best arena we currently have to test quantum gravity theories. The success of the inflationary paradigm in accounting for the observed inhomogeneities in the cosmic microwave background already illustrates this point to a certain extent because the paradigm is based on quantum field theory on the curved cosmological space-times. However, this analysis excludes the Planck era because the background space-time satisfies Einstein's equations all the way back to the big bang singularity. Using techniques from loop quantum gravity, the paradigm has now been extended to a self-consistent theory from the Planck regime to the onset of inflation, covering some 11 orders of magnitude in curvature. In addition, for a narrow window of initial conditions, there are departures from the standard paradigm, with novel effects, such as a modification of the consistency relation involving the scalar and tensor power spectra and a new source for non-Gaussianities. The genesis of the large scale structure of the universe can be traced back to quantum gravity fluctuations in the Planck regime. This report provides a bird's eye view of these developments for the general relativity community.

1 Introduction

In this conference, Professor Bicak and others described the ideas that Einstein developed in Prague during 1911-12. From then until 1915 he worked largely by himself on the grand problem of extending the reach of special relativity to encompass gravity. Finally, in November 1915, he provided us with the finished theory. For almost a century, the relativity community has been engaged in understanding the astonishingly rich physics the theory contains, testing it ever more accurately, and applying it to greater and greater domains of astrophysics and cosmology. The the-

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ory has so many marvelous features. Amazingly, the field equations turned out to provide an elliptic-hyperbolic system with a well-posed initial value problem. After many decades, we realized that the total mass of an isolated system is a well defined geometric invariant and, furthermore, positive if the local energy density of matter is positive. The theory naturally admits cosmological solutions in which the universe is expanding, just as the observations tell us. It admits black hole solutions that serve as engines for the most energetic phenomena seen in the universe. None of these fascinating features that we now regard as fundamental consequences were part of Einstein's motivation during his quest which he described as "*one of the most exciting and exacting times of my life*" [1]. He essentially handed to us the finished product on a platter. We have been engaged in uncovering the numerous hidden treasures it contains by working out the philosophical, mathematical, physical, astronomical and cosmological consequences of the new paradigm.

But we know that the theory is incomplete. Indeed, it exhibits its own fundamental limitations through singularities where space-time ends and general relativistic physics comes to a halt. We also understand that this occurs because general relativity ignores quantum physics. Perhaps the most outstanding example is the prediction of the big bang. If we go back in time, *much* before we reach the singularity, matter densities exceed the nuclear density, $\sim 10^{14} - 10^{15}$ gms/cc, where we definitely know that quantum properties of matter dominate. Since gravity couples to matter, the conceptual paradigm of general relativity becomes inadequate. If we go further back in time, general relativity presents us with an epoch in which densities reach $\sim 10^{94}$ gms/cc. This is the Planck scale and now physics of general relativity becomes inadequate not only conceptually but also in practice. In this regime we expect gross departures from Einstein's theory. Just as it is totally inadequate to use Newtonian mechanics to explore physics near the horizon of a solar mass black hole, it is incorrect to trust general relativity once the matter density and space-time curvature enter the Planck regime. Thus, *big bang is a prediction of general relativity in a domain in which it is simply invalid*. Normally physicists do not advertise such predictions of theories. But unfortunately they often seem to make an exception for the big bang. One hears statements like 'the cosmic microwave background (CMB) is a fingerprint of the big bang'. But in the standard scenario, CMB refers to a time some 380,000 years after the putative big bang. Existence or even the detailed features of CMB have no bearing on whether the big bang with *infinite* matter density and curvature ever occurred. Indeed, as we will see, loop quantum cosmology (LQC) has no big bang singularity and yet reproduces these features. What about inflation? In the standard scenario, it is supposed to have commenced 'only' 10^7 Planck seconds after the big bang. Does its success not imply that there was a big bang? It does *not* because the matter density and curvature at the onset of inflation are only $10^{-11} - 10^{-12}$ times the Planck scale. Indeed, this is why one can use Einstein's equations and quantum field theory (QFT) on Friedmann, Lemaître, Robertson, Walker (FLRW) solutions in the analysis of inflation. Inflationary physics by itself cannot say what really happened in the Planck regime and, again, as we will see, is compatible with the LQC prediction that there was no big bang singularity.

Thus, to know what really happened in the Planck regime and go beyond the singularities predicted by general relativity, we need a viable quantum theory of gravity. Since the search for this theory has been ongoing for decades, justifiably, there is sometimes a sentiment of pessimism in the general relativity circles. In my view, this is largely because one judges progress using the criterion of general relativity. In a masterful stroke, Einstein gave us the final theory and we have been happily engaged in investigating its content. It seems disappointing that this has not happened with quantum gravity. But progress of physical theories has more often mimicked the development of quantum theory rather than general relativity. More than a century has passed since Planck's discovery that launched the quantum. Yet, the theory is incomplete. We do not have a satisfactory grasp of the foundational issues, often called the 'measurement problem', nor do we have a single example of an interacting QFT in 4 dimensions. A far cry from what Einstein offered us in 1915! Yet, no one would deny that quantum theory has been extremely successful; indeed, much more so than general relativity.

Thus, while it is tempting to wait for another masterful stroke like Einstein's to deliver us a finished quantum gravity theory, it is more appropriate to draw lessons from quantum theory. There, progress occurred by focussing not on the 'final, finished' theory, but on concrete physical problems where quantum effects were important. It would be more fruitful to follow this path in quantum gravity. Indeed, even though we are far from a complete theory, advances can occur by focusing on specific physical problems and challenges.

Over the last several years, research in loop quantum gravity (LQG) has been driven by this general philosophy. In addition to seeking a completion of the general program based on connection variables, spin networks and spin foams, more and more effort is now focused on specific physical problems where quantum gravity effects are expected to be important. The idea behind this research is to *first truncate general relativity (with matter) to sectors tailored to specific physical problems, and then pass to quantum theory using the background independent methods based on the specific quantum geometry that underlies LQG*. This strategy of focusing on specific problems of quantum gravity also distinguishes LQG from string theory in terms of their main trust in the last few years. In string theory, the focus has shifted to *using* the well-understood parts of gravity to explore other areas of physics —use of the AdS/CFT hypothesis to understand the strong coupling regime of QCD, to gain new insights into hydrodynamics and tackle the strong coupling problems in mathematical physics to better understand condensed matter systems such as high temperature super-conductivity. The LQG community, on the other hand, has continued to tackle the long standing problems of quantum gravity per se —absence of a space-time in the background, the problem of time, fate of cosmological singularities in the quantum theory, quantum geometry of horizons, and derivation of the graviton propagator in a background independent setting.

The goal of my talk was to report the advances in the cosmology of the very early universe that have resulted from a continued application of the truncation strategy in LQG. Of course, both the talk and this report can only provide a bird's eye view of these developments. The results I reported are based largely on joint work with

Alejandro Corichi, Tomasz Pawłowski and Parampreet Singh [2, 3, 4, 5, 6, 7] on the singularity resolution in cosmology; with David Sloan [8, 9] on effective LQC dynamics tailored to inflation, with Wojciech Kaminski and Jerzy Lewandowski [10] on QFT on quantum space-times; and especially with Ivan Agullo and William Nelson on extension of the cosmological perturbation theory to the Planck regime and its application to inflation [11, 12, 13]. Therefore, there is a large overlap with the material covered in these original references. Finally, by now there are well over a 1000 papers on LQC which include several investigations of inflationary dynamics. What I can cover constitutes only a very small fraction of what is known. For reviews on results until about a year ago, see, e.g. [14, 15].

2 Setting the Stage

Perhaps the most significant reason behind the rapid and spectacular success of quantum theory, especially in its early stage, is the fact that there was already a significant accumulation of relevant experimental data, and further experiments to weed out ideas could be performed on an ongoing basis. Unfortunately this is *not* the case for quantum gravity simply because theory has raced far ahead of technology. Indeed, even in the classical regime, we still lack detailed tests of general relativity in the strong field regime!

Currently, the early universe offers by far the best arena to test various ideas on quantum gravity. Most scenarios assume that the early universe is well described by a FLRW solution to Einstein's equations with suitable matter, *together with* first order perturbations. The background is treated classically, as in general relativity, and the perturbations are described by *quantum fields*. Thus, the main theoretical ingredient in the analysis are: cosmological perturbation theory and QFT on FLRW space-times. It is fair to say that among the current scenarios, the inflationary paradigm has emerged as the leading candidate. In addition to the common assumption described above, this scenario posits:

- Sometime in its early history, the universe underwent a phase of rapid expansion. This was driven by the slow roll of a scalar field in a suitable potential causing the Hubble parameter to be nearly constant.
- Fourier modes of the quantum fields representing perturbations were initially in a specific state, called the Bunch-Davies (BD) vacuum, for a certain set of co-moving wave numbers $(k_o, 2000k_o)$ where the physical wave length of the mode k_o equals the radius R_{LS} of the observable universe at the surface of last scattering.¹

¹ Strictly speaking, the BD vacuum refers to deSitter space; it is the unique 'regular' state which is invariant under the full deSitter isometry group. During slow roll, the background FLRW geometry is only approximately deSitter whence there is some ambiguity in what one means by the BD vacuum. One typically assumes that all the relevant modes are in the BD state (tailored to) a few e-foldings before the mode k_o leaves the Hubble horizon. Throughout this report, by BD vacuum I mean this state.

- Soon after any mode exits the Hubble radius, its quantum fluctuation can be regarded as a classical perturbation and evolved via linearized Einstein's equations.

One then evolves the perturbations from the onset of the slow roll till the end of inflation using QFT on FLRW space-times and calculates the power spectrum (see, e.g., [16, 17, 18, 19, 20]). When combined with standard techniques from astrophysics to further evolve the results to the surface of last scattering, one finds that they are in excellent agreement with the inhomogeneities seen in the CMB. Supercomputer simulations have shown that these inhomogeneities serve as seeds for the large scale structure in the universe. Thus, in a precise sense, the origin of the qualitative features of the observed large scale structure can be traced back to the fluctuations in the quantum vacuum at the onset of inflation. This is both intriguing and very impressive.

Over the years, the inflationary paradigm has witnessed criticisms from the relativity community, most eloquently expressed by Roger Penrose (see, e.g., [21]). However, these criticisms refer to the motivations that were originally used by the proponents, rather than to the methodology underlying its success in accounting for the CMB inhomogeneities. There are plenty of examples in fundamental physics where the original motivations turned out not to be justifiable but the idea was highly successful. I share the view that, while the basic assumptions, listed above, have not been justified from first principles, the success of the inflationary paradigm with CMB measurements is impressive because one 'gets much more out than what one puts in'.

In spite of this success, however, the inflationary scenario is conceptually incomplete in several respects. (For a cosmology perspective on these limitations see e.g. [22].) In particular, as Borde, Guth and Vilenkin [23] showed, inflationary space-times inherit the big-bang singularity in spite of the fact that the inflaton violates the standard energy conditions used in the original singularity theorems [24]. As we discussed in section 1, this occurs because one continues to use general relativity even in the Planck regime in which it is simply not applicable. One expects new physics to play a dominant role in this regime, thereby resolving the singularity and significantly changing the very early history of the universe. One is therefore led to ask: Will inflation arise naturally in the resulting deeper theory? Or, more modestly, can one at least obtain a consistent quantum gravity extension of this scenario?

The open-ended nature of the inflationary paradigm has three facets. First, there are issues whose origin lies in particle physics. Where does the inflaton come from? How does potential arise? Is there a single inflaton or many? If many, what are the interactions between them? Since the required mass of the inflaton is very high, above 10^{12} GeV, the fact that we have not seen it at CERN does not mean it cannot exist. But in the inflationary scenario this is the only matter field in the early universe and particles of the standard model are supposed to be created during 'reheating' at the end of inflation when the inflaton is expected to roll back and forth around its minimum. However, how this happens is not at all well-understood. What are the admissible interactions between the inflaton and the standard model particles which causes this decay? Does the decay produce the correct abundance of the standard model particles? These questions with origin in particle physics are wide open.

The second issue is the quantum to classical transition referred to in the last assumption of standard inflation. In practice one calculates the expectation values of perturbations and the two point function at the end of inflation and assumes one can replace the actual quantum state of perturbations with a Gaussian statistical distribution of classical perturbations with the mean and variance given by the quantum expectation value and the 2-point function. As a calculational device this strategy works very well. However, what happens physically? While this issue has drawn attention, we do not yet have a clear consensus on the actual, detailed physics that is being approximated in the last assumption.

The third set of issues have their origin in quantum gravity. In the standard inflationary scenario, one specifies initial conditions at the onset of inflation and then evolves the quantum perturbations. As a practical strategy, something like this is unavoidable within general relativity. Ideally one would like to specify the initial conditions at ‘the beginning’, but one simply cannot do this because the big bang is singular. Furthermore, since the curvature at the onset of inflation is some $10^{-11} - 10^{-12}$ times the Planck scale, by starting calculations there, one bypasses the issue of the correct Planck scale physics. But this is just an astute stopgap measure. Given any candidate quantum gravity theory, one can and *has to* ask whether one can do better. Can one meaningfully specify initial conditions in the Planck regime? In a viable quantum gravity theory, this should be possible because there would be no singularity and the Planck scale physics would be well-controlled. If so, in the systematic evolution from there, does a slow roll phase compatible with the 7 year WMAP data [25] arise *generically* or is an enormous fine tuning needed? One could argue that it is acceptable to use fine tuning because, after all, the initial state is very special. If so, can one provide physical principles that select this special state? In the standard inflationary scenario, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on *quantum* cosmological space-times needed to adequately handle physics at that stage? Can one *arrive at* the BD vacuum (at the onset of the WMAP slow roll) starting from the initial conditions at the Planck scale?

In this report, I will not address the first two sets of issues. Rather, the focus will be on the incompleteness related to the third set, i.e., on quantum gravity. Systematic advances within LQC over the past six years have provided a viable extension of the inflationary scenario all the way to the Planck regime. This extension enables us to answer in detail most of the specific questions posed above. To arrive at a coherent extension, LQC had to develop a conceptual framework, mathematical tools and high precision numerical simulations because the issues are so diverse: The meaning of time in the Planck regime; the nature of quantum geometry in the cosmological context; QFT on *quantum* cosmological space-times; renormalization and regularization of composite operators needed to compute stress energy and back reaction; and, relation between theory and the WMAP data.

A consistent theoretical framework to deal with cosmological perturbations on quantum FLRW space-times now exists [12]. Starting with ‘natural’ initial conditions in the Planck regime, one can evolve the quantum perturbations on quantum FLRW backgrounds and study in detail the pre-inflationary dynamics [11, 13]. De-

tailed numerical simulations have shown that the *predictions are in agreement with the power spectrum and the spectral index reported in the 7 year WMAP data*. However, there is also a small window in the parameter space where the initial state at the onset of inflation differs sufficiently from the BD vacuum assumed in standard inflation to give rise to new effects. These are the prototype observable signatures of pre-inflationary dynamics. In this sense, *LQC offers the possibility of extending the reach of cosmological observations to the deep Planck regime of the early universe*.

3 Why Pre-inflationary Dynamics Matters

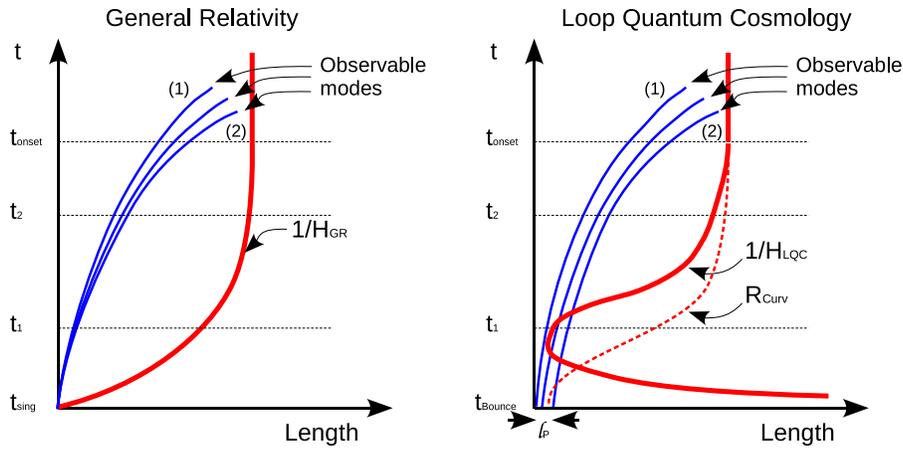


Fig. 1 Schematic time evolution of the Hubble radius (red solid line on the right in each panel) and of wave lengths of three modes seen in the CMB (three solid blue lines in each panel). Credits: W. Nelson. *Left Panel: General relativity*. The modes of interest have wave lengths less than the Hubble radius $1/H_{GR}$ all the way from the big bang (t_{sing}) until after the onset of slow roll. *Right Panel: LQC*. The Hubble radius diverges at the big bounce (t_{Bounce}), decreases rapidly to reach its minimum in the deep Planck era and then increases monotonically. Because of this, modes seen in the CMB can have wave lengths larger than the Hubble radius $1/H_{LQC}$ in the very early universe. Detailed analysis shows that what really matters is the curvature radius R_{Curv} shown schematically by the dashed red line rather than the Hubble radius $1/H_{LQC}$. But again the modes can exit the curvature radius in the Planck regime and, if they do, they are excited during the pre-inflationary evolution. They will not be in the BD vacuum at the onset of slow roll inflation.

It is often claimed that pre-inflationary dynamics will not change the observable predictions of the standard inflationary scenario. Indeed, this belief is invoked to justify why one starts the analysis just before the onset of the slow roll. The belief stems from the following argument, sketched in the left panel of Fig. 1. If one evolves the modes that are seen in the CMB *back* in time starting from the onset

of slow roll, their physical wave lengths λ_{phy} continue to remain within the Hubble radius $1/H_{\text{GR}}$ all the way to the big bang. Therefore, one argues, they would not experience curvature and their dynamics would be trivial all the way from the big bang to the onset of inflation; because they are not ‘exited’, all these modes would be in the BD vacuum at the onset of inflation. However, this argument is flawed on two accounts. First, if one examines the equation governing the evolution of these modes, one finds that what matters is the curvature radius $R_{\text{curv}} = \sqrt{6/\mathfrak{R}}$ determined by the Ricci scalar \mathfrak{R} , and not the Hubble radius. The two scales are equivalent only during slow roll on which much of the intuition in inflation is based. However, in general they are quite different from one another. Thus we should compare λ_{phy} with R_{curv} in the pre-inflationary epoch. The second and more important point is that the pre-inflationary evolution should not be computed using general relativity, as is done in the argument given above. One has to use an appropriate quantum gravity theory since the two evolutions are expected to be *very* different in the Planck epoch. Then modes that are seen in the CMB could well have $\lambda_{\text{phy}} \gtrsim R_{\text{curv}}$ in the pre-inflationary phase. If this happens, these modes *would be* excited and the quantum state at the onset of the slow roll could be quite different from the BD vacuum. Indeed, the difference could well be so large that the amplitude of the power spectrum and the spectral index are incompatible with WMAP observations. In this case, that particular quantum gravity scenario would be ruled out. On the other hand, the differences could be more subtle: the new power spectrum for scalar modes could be compatible with observations but there may be departures from the standard predictions that involve tensor modes or higher order correlation functions of scalar modes, changing the standard conclusions on non-Gaussianities [26, 27, 28, 29]. In this case, the quantum gravity theory would have interesting predictions for future observational missions. Thus, pre-inflationary dynamics can provide an avenue to confront quantum gravity theories with observations.

These are not just abstract possibilities. The right panel of Fig. 1 shows schematically the situation in LQC. (For the precise behavior obtained from numerical simulations, see Fig. 1 in [13].) The wave lengths of some of the observable modes *can* exit the curvature radius during pre-inflationary dynamics, whence there are departures from the standard predictions (which turn out to be of the second type in the discussion above).

So far we have focused only on why a common argument suggesting that pre-inflationary dynamics cannot have observational consequences is fallacious. At a deeper level, pre-inflationary dynamics matters because of a much more general reason: It is important to know if inflationary paradigm is part of a conceptually coherent framework encompassing the quantum gravity regime. Can one trust the standard scenario in spite of the fact that the modes it focuses on become trans-Planckian in the pre-inflationary epoch? Does one have to artificially fine-tune initial conditions in the Planck regime to arrive at the BD vacuum? Do initial conditions for the background in the Planck regime naturally give rise to solutions that encounter the desired inflationary phase some time in the future evolution? To investigate any one of these issues, one needs a reliable theory for pre-inflationary dynamics and also good control on its predictions.

4 The LQG Strategy

LQG offers an attractive framework to investigate pre-inflationary dynamics because its underlying quantum geometry becomes important at the Planck scale and leads to the resolution of singularities in a variety of cosmological models. In particular the following cosmologies have been investigated in detail: the $k=0$ and $k=1$ FLRW models are discussed in [30, 31, 2, 3, 4, 5, 32, 33, 34]; a non-zero cosmological constant is included in [35, 36, 6]; anisotropic are discussed via Bianchi I, II and IX models in [37, 38, 39, 40]; and the inhomogeneous Gowdy models, which have been analyzed in detail in classical general relativity, have been studied in [41, 42, 43, 44, 45]. In all cases, the big bang singularity is resolved and replaced by quantum bounces. It is therefore natural to use LQC as the point of departure for extending the cosmological perturbation theory.

In the standard perturbation theory, one begins with linearized solutions of Einstein's equations on a FLRW background. Unfortunately, we cannot mimic this procedure because in LQG we do not yet have the analog of full Einstein's equations that one would have to perturb. But one can adopt the *truncation strategy* discussed in section 1. Thus, one starts with a truncation Γ_{Trun} of the phase space Γ of general relativity, tailored to the linear perturbations off FLRW backgrounds. Furthermore since we are interested in the issue of whether the inflationary framework admits a quantum gravity extension, the matter source will be just a scalar field ϕ with the simplest, i.e. quadratic, potential $V(\phi) = (1/2)m^2\phi^2$. Thus, Γ_{Trun} is given by $\Gamma_{\text{Trun}} = \Gamma_o \times \Gamma_1$ where Γ_o is the 4-dimensional FLRW phase space, with the scale factor a and the homogeneous inflaton ϕ as configuration variables, and Γ_1 is the phase space of gauge invariant first order perturbations consisting of a scalar mode and two tensor modes. Since the background fields are homogeneous, it is simplest to assume that the perturbations are purely inhomogeneous. Thus, regarded as a sub-manifold of the full phase space Γ , Γ_{Trun} is the normal bundle over Γ_o .

As usual, for perturbations one can freely pass between real space and momentum space using Fourier transforms of fields in co-moving coordinates. For pre-inflationary dynamics, we work with the Mukhanov-Sasaki variables, denoted by $\mathcal{Q}_{\mathbf{k}}$, because they are well-defined all the way from the bounce to the onset of slow roll.² We denote the two tensor modes collectively by $\mathcal{T}_{\mathbf{k}}$. This structure is the same as that used in standard inflation [46].

New features appear in the next step: In the passage to quantum theory, we work with the *combined system*, i.e., with all of Γ_{Trun} . Therefore, we are naturally led a theory in which not only the perturbations but even the background geometry is quantum. Rather than having quantum fields $\hat{\mathcal{Q}}$ and $\hat{\mathcal{T}}$ propagating on a classical FLRW space-time, they now propagate on a *quantum* FLRW geometry.

² The curvature perturbations $\mathcal{R}_{\mathbf{k}}$ fail to be well-defined at the 'turning point' where $\dot{\phi} = 0$, which occurs during pre-inflationary dynamics. However, they are much more convenient for relating the spectrum of perturbations at the end of inflation with the CMB temperature fluctuations. Therefore, we first calculate the power spectrum $\mathcal{P}_{\mathcal{Q}}$ for Mukhanov-Sasaki variable $\mathcal{Q}_{\mathbf{k}}$ and then convert it to $\mathcal{P}_{\mathcal{R}}$, reported in Fig. 3.

Thus, the strategy to truncate the classical phase space and then pass to quantum theory using LQG techniques leads to a novel quantum theory. The total Hilbert space is a tensor product, $\mathcal{H} = \mathcal{H}_0 \otimes \mathcal{H}_1$, where \mathcal{H}_0 is the space of wave functions Ψ_0 describing a quantum FLRW geometry and \mathcal{H}_1 is the space of quantum states ψ of perturbations. The first task is to construct the Hilbert space \mathcal{H}_0 of physical states $\Psi_0(a, \phi)$, by imposing the Hamiltonian constraint on the homogeneous sector Γ_0 . The second task is to study quantum dynamics of fields $\hat{\mathcal{Q}}$ and $\hat{\mathcal{T}}$ on the quantum geometry encapsulated in $\Psi_0(a, \phi)$. In particular we have to introduce the Hilbert space \mathcal{H}_1 of wave functions $\psi(\mathcal{Q}_{\mathbf{k}}, \mathcal{T}_{\mathbf{k}})$ of perturbations and develop techniques to calculate the 2-point functions on \mathcal{H}_1 that are needed to obtain the scalar and the tensor power spectra. The final task is to check the self-consistency of the truncation strategy with which we began. Already in the classical theory, the truncated phase space Γ_{Trun} is useful only so long as the back reaction can be neglected. Therefore, in the quantum theory, we have to check that the \mathcal{H} admits solutions $\Psi_0 \otimes \psi$ in which the energy density of perturbations is negligible compared to that in the background all the way from the LQC bounce to the onset of slow roll. On the analytical side, this requires the introduction of suitable regularization and renormalization techniques for quantum fields $\hat{\mathcal{Q}}$ and $\hat{\mathcal{T}}$ propagating on the quantum background Ψ_0 . On the numerical side, one has to devise accurate numerical methods to calculate the energy density in perturbations with sufficient precision during the evolution all the way from the bounce to the onset of inflation, as the background energy density falls by some 11 orders of magnitude.

These tasks have been carried out in [11, 12, 13] using earlier results obtained in [4, 5, 8, 9, 10, 6, 7]. The next two sections provide a flavor of this analysis.

5 Analytical Aspects

- *Background Quantum Geometry:* In the classical theory, dynamics on Γ_0 is generated by the single, homogeneous, Hamiltonian constraint, $\mathbb{C}_0 = 0$. Each dynamical trajectory on Γ_0 represents a classical FLRW space-time. In quantum theory, physical states are represented by wave functions $\Psi_0(a, \phi)$ satisfying the quantum constraint $\hat{\mathbb{C}}_0 \Psi_0 = 0$. Each of these solutions represents a quantum FLRW geometry.

We are interested in those solutions Ψ_0 which remain sharply peaked on classical FLRW solutions at late times. In the sector of the theory that turns out to be physically most interesting [13], these states remain sharply peaked all the way up to the bounce but in the Planck regime they follow certain effective trajectories which include quantum corrections [15, 7]. In particular, rather than converging on the big bang singularity, as classical FLRW solutions do, they exhibit a bounce when the density reaches $\rho_{\text{max}} \approx 0.41\rho_{\text{Pl}}$ (see Fig. 2). It turns out that each (physically distinct) effective solution is completely characterized by the value ϕ_{B} that the inflaton assumes at the bounce. *This value turns out to be the key free parameter of the theory.* Finally, we need full quantum evolution from the bounce only until the density and curvature fall by a factor of, say, $10^{-6} - 10^{-7}$. After that, the background can be

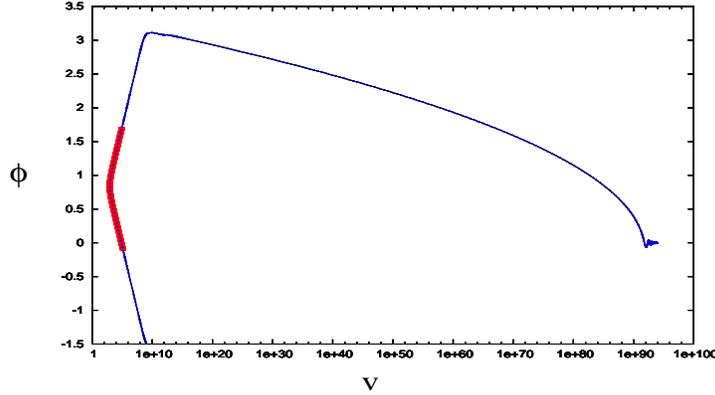


Fig. 2 An effective LQC trajectory in presence of an inflation with a quadratic potential $(1/2)m^2\phi^2$, where the value $m = 6.1 \times 10^{-6}m_{\text{Pl}}$ of the mass is calculated from the 7 year WMAP data (source [7]). Here $V \sim a^3$ is the volume of a fixed fiducial region. The long (blue) sloping line at the top depicts slow roll inflation. As V decreases (right to left), we go back in time and the inflaton ϕ first climbs up the potential, then turns around and starts going descending. In classical general relativity, volume would continue to decrease until it becomes zero, signalling the big bang singularity. In LQC, the trajectory bounces at $\phi \sim 0.95$ and volume never reaches zero; the entire evolution is non-singular.

taken to follow the general relativity trajectory to a truly excellent approximation.³ (For details, see [4, 5, 7]).

- *Dynamics of Perturbations:* There is an important subtlety which is often overlooked in the quantum gravity literature: Dynamics of perturbations is *not* generated by a constraint, or, indeed by *any* Hamiltonian. On the truncated phase space Γ_{Trun} , the dynamical trajectories are tangential to a vector field X^α of the form $X^\alpha = \Omega_o^{\alpha\beta} \partial_\beta C_o + \Omega_1^{\alpha\beta} \partial_\beta C'_2$ where Ω_o and Ω_1 are the symplectic structures on Γ_o and Γ_1 , and C'_2 is the part of the second order Hamiltonian constraint function in which only terms that are quadratic in the first order perturbations are kept (ignoring terms which are linear in the second order perturbations). X^α fails to be Hamiltonian on Γ_{Trun} because C'_2 depends not only on perturbations but also background quantities. However, given a dynamical trajectory $\gamma_o(t)$ on Γ_o and a perturbation at a point thereon, X^α provides a canonical lift of $\gamma_o(t)$ to the total space Γ_{Trun} , describing the evolution of that perturbation along $\gamma_o(t)$.

Therefore, in the quantum theory, dynamics of the combined system *cannot be obtained by simply imposing a quantum constraint* on the wave functions $\Psi_o \otimes \psi$ of the combined system. One has to follow a procedure similar to what is done in the classical theory. Thus, one first obtains a background quantum geometry Ψ_o

³ During this phase, the scalar field is monotonic in time in the effective trajectory. Therefore we can use the scalar field as an ‘internal’ or ‘relational’ time variable with respect to which the background scale factor (and curvature) as well as perturbations evolve. This interpretation is not essential but very helpful in practice because of the form of the Hamiltonian constraint $\hat{C}_o \Psi_o = 0$ (for details, see e.g. [15]).

by solving $\hat{C}_o \Psi_o(a, \phi) = 0$, specifies the quantum state $\psi(\mathcal{Q}_k, \mathcal{T}_k)$ of the perturbation at, say, the bounce time, and evolves it using the operator \hat{C}'_2 . The resulting state $\Psi(a, \mathcal{Q}_k, \mathcal{T}_k, \phi)$ describes the evolution of the quantum perturbations ψ on the quantum geometry Ψ_o in the Schrödinger picture. (For details, see [12]).

- *Trans-Planckian Issues:* Quantum perturbations $\hat{\mathcal{L}}, \hat{\mathcal{T}}$ propagate on quantum geometries Ψ_o which are all regular, free of singularities. Thus, the *the framework is tailored to cover the Planck regime*. What is the status of the ‘trans-Planckian problems’ which are associated with modes of trans-Planckian frequencies in heuristic discussions? To probe this issue one has to first note that the quantum Riemannian geometry underlying LQG is quite subtle [47, 48, 49]: in particular, while there is a minimum non-zero eigenvalue of the area operators, the area gap, there is no volume gap, even though their eigenvalues are also discrete [50, 51].⁴ As a consequence, there is no fundamental obstacle preventing the existence trans-Planckian modes of perturbations in our truncated theory. Indeed, in the homogeneous LQC models that have been analyzed in detail, the momentum $p_{(\phi)}$ of the scalar field ϕ is generally *huge* in Planck units. This poses no problem and, in particular, on the physical Hilbert space the total energy density is still guaranteed to be bounded by ρ_{\max} (see, e.g. [15]). Similarly, perturbations $\hat{\mathcal{L}}, \hat{\mathcal{T}}$ of our truncated theory are permitted to acquire trans-Planckian momenta. The real danger is rather that, in presence of such modes, the *energy density* in perturbations may fail to be negligible compared to that in the quantum background geometry. This issue is extremely non-trivial, especially in the Planck regime. If the energy density does become comparable to that in the background, then we would not be able to neglect the back-reaction and our truncation would fail to be self-consistent.⁵ *This is the trans-Planckian problem we face in our theory of quantum perturbations on inflationary quantum geometries.* To address it we need regularization and renormalization methods to compute energy density for quantum fields on quantum FLRW geometries. (For details, see [12, 13]).

- *An Unforeseen Simplification:* As we just noted, the underlying FLRW quantum geometry provides the necessary control on calculations in the deep Planck regime. However, it confronts us with a new challenge of developing the mathematical theory of quantum fields on quantum geometries. At first this problem seems formidable. But fortunately there is a key simplification within the test field approximation we are using in the truncated theory [10, 12]: Mathematically the evolution of $\hat{\mathcal{L}}, \hat{\mathcal{T}}$ on any one of our quantum geometries Ψ_o is completely equivalent to that of these fields propagating on a dressed, effective metric \tilde{g}_{ab} constructed from Ψ_o .⁶ Note that \tilde{g}_{ab} contains quantum corrections and does not satisfy Einstein’s equation.

⁴ Properties of the eigenvalues of length operators [52, 53, 54] have not been analyzed in comparable detail. But since their definitions involve volume operators, it is expected that there would be no ‘length gap’.

⁵ Of course, this would not imply that the inflationary scenario does not admit an extension to the Planck regime. But to obtain it one would then have to await the completion of a *full* quantum gravity theory.

⁶ For scalar modes, the classical equation of motion involves also ‘an external potential’ \mathfrak{A} . This has also to be replaced by a dressed effective potential $\tilde{\mathfrak{A}}$. for details, see [13].

Indeed, it does not even satisfy the effective equations of LQC because, whereas the effective trajectories follow the ‘peak of Ψ_o ’, \tilde{g}_{ab} also knows about certain fluctuations encoded in Ψ_o .⁷ Nonetheless, since \tilde{g}_{ab} is a smooth metric with FLRW symmetries, it is now possible to use the rich machinery of QFT on cosmological space-times to analyze the dynamics of $\hat{\mathcal{Q}}, \hat{\mathcal{T}}$ in detail. In addition, one can now make use of the powerful technique of adiabatic regularization that has been developed over some three decades [55, 56, 57, 58, 59, 60]. In particular, by restricting ourselves to states ψ of perturbations which are of 4th adiabatic order, one can compute the expectation values of energy density. This provides a clear avenue to face the true trans-Planckian problem, i.e., to systematically test if the truncation approximation is valid.

This remarkable simplification occurs because the dynamics of test quantum fields is not sensitive to all the details of the probability amplitude for various FLRW metrics encapsulated in Ψ_o ; it experiences only to a few moments of this distribution. The phenomenon is analogous to the propagation of light in a medium where all the complicated interactions of the Maxwell field with the atoms in the medium can be captured just in a few parameters such as the refractive index. (For details, see [10, 12, 13]).

- *Initial Conditions:* In the Schrödinger picture, the above simplification enables us to evolve the quantum state ψ of perturbations. But we still have to specify the initial conditions. Since the big bang of general relativity is replaced by the big bounce in LQC, it is natural to specify them at the bounce. Now, in the truncation approximation, perturbation is treated as a test field. Therefore, it is appropriate to assume that the initial state has the form $\Psi_o \otimes \psi$ at the bounce. Furthermore this tensor product form will be preserved under dynamics so long as the back reaction due to the perturbation remains negligible.

Let us begin with Ψ_o . In the effective theory, phase space variables are subject to certain constraints at the bounce. We assume that Ψ_o is sharply peaked at a point on this constraint surface (with small fluctuations in each of the two ‘conjugate’ variables). At the bounce, the allowed range of ϕ is finite but large, $|\phi_B| \in (0, 7.47 \times 10^5)$ in Planck units, and a detailed analysis of effective solutions has shown that unless $|\phi_B| < 0.93$, the effective trajectory necessarily encounters a slow roll phase compatible with WMAP sometime in the future [9]. Thus, the peak of initial Ψ_o is almost unconstrained. However, the requirement that Ψ_o be peaked is very strong and makes the initial state of background geometry very special.

For perturbations, we assume the following three conditions on ψ at the bounce: i) Symmetry: ψ should be invariant under the FLRW isometry group, i.e., under spatial translations and rotations. This condition is natural because these are the symmetries of the background Ψ_o and hence also of \tilde{g}_{ab} it determines; ii) Regularity: ψ should be of 4th adiabatic order so that the Hamiltonian operator has a well-defined action on it; and, iii) The initial renormalized energy density $\langle \psi | \hat{\rho} | \psi \rangle_{\text{ren}}$

⁷ While this difference is conceptually important, because the states Ψ_o of interest are so sharply peaked, in practice the deviations from effective trajectories are small even in the Planck regime. Of course the deviations from classical solutions are enormous in the Planck regime because \tilde{g}_{ab} is non-singular.

in the perturbation should be negligible compared to the energy density ρ_{\max} in the background. We have an explicit example, $|\psi\rangle = |0_{\text{obv}}\rangle$, of such a state called the ‘obvious vacuum of 4th adiabatic order’ which has several attractive properties [13]. Furthermore we also know that, given a state satisfying these properties, there are ‘infinitely many’ such states in its neighborhood. Thus, the existence of the desired states is assured. However, in view of the large freedom that remains, it is desirable to develop clear-cut physical criteria to cut down this freedom significantly. This is an open issue, currently under investigation. (For details, see [11, 12, 13]).

Let us summarize the analytical framework. The initial condition for the quantum state $\Psi_o \otimes \psi$ of the combined system is specified at the bounce in such a manner that a slow roll inflation compatible with the 7 year WMAP data is guaranteed in the background geometry. Thanks to an unforeseen simplification, we can use techniques from QFT on cosmological space-times to evolve the perturbations $\hat{\mathcal{Q}}$ and $\hat{\mathcal{F}}$ on the quantum background geometry Ψ_o . Finally, the initial conditions guarantee that the truncation approximation does hold at the bounce: ψ can be regarded as a perturbation whose back reaction on Ψ_o is negligible initially. Furthermore, states are sufficiently regular to enable us to calculate the energy density in the background and in the perturbation at all times. Therefore, one can carry out the entire evolution numerically, calculate the power spectra and spectral indices and check if the truncation approximation continues to hold under evolution all the way from the bounce to the onset of the slow roll.

As discussed in section 3, a priori there are several possible outcomes. Pre-inflationary dynamics could have such a strong effect that the power spectra and the spectral indices that result from these calculations are incompatible with the WMAP observations. In this case, the LQC extension would be ruled out by observations. It is also possible that the Planck scale dynamics is such that the back reaction ceases to be negligible very soon after the bounce making the truncation strategy inconsistent. In this case, our truncation strategy would fail to be self-consistent. Finally, even if these possibilities do not occur, we may find that, for observable modes, the state at the onset of inflation is sufficiently different from the BD vacuum that there are departures from the standard inflationary predictions for future observations. One needs explicit numerical simulations to find out which of these various a priori possibilities are realized.

6 Numerical Aspects, Observations and Self-Consistency

In this section, numerical values of all physical quantities will be given in natural Planck units $c=\hbar=G=1$ (as opposed to the reduced Planck units used in the cosmology literature where one sets $8\pi G=1$). We will use both the conformal time $\tilde{\eta}$ and the proper (or cosmic) time \tilde{t} determined by the dressed effective metric \tilde{g}_{ab} via $d\tilde{s}^2 := \tilde{g}_{ab}dx^a dx^b = a^2(-d\tilde{\eta}^2 + d\mathbf{x}^2) = -d\tilde{t}^2 + a^2 d\mathbf{x}^2$ (where, as usual, x^a are the co-moving coordinates). This is because the cosmology literature generally uses conformal time but comparison with general relativity can be made more transpar-

ent in cosmic time by setting it equal to zero at the big bang in general relativity and at the big bounce in LQC.

$\phi(t_B)$	k_*	$\ln k_*$	$\lambda_*(t_B)$	t_{k_*}	$\ln[a(t_{k_*})/a(t_B)]$
0.934	0.0016	-6.4	4008	1.8×10^5	5.2
1	0.024	-3.7	261	5.2×10^5	8.0
1.05	0.17	-1.8	37.1	7.6×10^5	10
1.1	1.2	0.2	5.1	1.0×10^6	12
1.15	9.17	2.83	0.63	1.25×10^6	13.9
1.2	70.7	4.2	0.09	1.48×10^6	16
1.3	4.58×10^3	8.43	1.36×10^{-3}	1.97×10^6	20.2
1.5	2.7×10^7	17.1	2.3×10^{-7}	2.9×10^6	28.9

Table 1 This table from [13] shows the value of the reference co-moving momentum k_* used in the WMAP data, the corresponding physical wavelength $\lambda_*(\tilde{t}_B)$ at the bounce, the time $\tilde{t}(k_*)$ at which the mode k_* exits the Hubble radius during inflation, and $\ln[a(\tilde{t}(k_*))/a(\tilde{t}_B)]$, the number of e-folds of expansion between the bounce and $\tilde{t}(k_*)$. We focus on the range for ϕ_B that is relevant to explore whether pre-inflationary dynamics can lead to deviations from the BD vacuum at the onset of the slow roll.

- *WMAP Phenomenology:* The 7 year WMAP data [25] uses a reference mode $k_* \approx 8.58k_o$ where, as before, k_o is the co-moving wave number of the mode whose physical wave length equals the radius of the observable universe at the surface of last scattering. The WMAP analysis provides us with the amplitude $\mathcal{P}_{\mathcal{R}}(k_*)$ of the power spectrum and the spectral index $n_s(k_*)$ which encodes the small deviation from scale invariance, both for the scalar perturbations. The values are given by

$$\mathcal{P}_{\mathcal{R}}(k_*) = (2.430 \pm 0.091) \times 10^{-9} \quad \text{and} \quad n_s(k_*) = 0.968 \pm 0.012. \quad (1)$$

For the quadratic potential considered here, these observational data provide the following values of the Hubble parameter H and the slow roll parameter $\varepsilon = -\dot{H}/H^2$:

$$H(\tilde{\eta}(k_*)) = 7.83 \times 10^{-6} \quad \text{and} \quad \varepsilon(\tilde{\eta}(k_*)) = 8 \times 10^{-3}. \quad (2)$$

where $\tilde{\eta}(k_*)$ is the conformal time in our dressed effective metric \tilde{g}_{ab} at which the mode k_* exited the Hubble radius and the ‘dot’ refers to the derivative w.r.t. \tilde{t} . Since the physical wave length of the mode k_o is 8.58 times larger, it must have left the Hubble radius ~ 2 e-foldings before $\tilde{\eta}(k_*)$. Onset of slow roll inflation is taken to commence a little before the k_o exits its Hubble horizon. The value of the Hubble parameter at this time is so low that the total energy density is less than $10^{-11}\rho_{\text{Pl}}$. Therefore throughout the inflationary era general relativity is an excellent approximation to LQC. Equations of general relativity (or, LQC) determine the mass m of the inflaton as well as values of the inflaton ϕ at $\tilde{\eta}(k_*)$:

$$m = 1.21 \times 10^{-6} \quad \text{and} \quad \phi(\tilde{t}(k_*)) = \pm 3.15. \quad (3)$$

Because of the observational error bars, these quantities are uncertain by about 2%. In the numerical simulations we use the value of m given in (3). (For details, see [9]).

- *Evolution of the Background:* So far numerical evolutions of the background wave function Ψ_o are feasible only for kinetic dominated bounces, i.e., bounces for which ϕ_B is small. This is because the required time over which one has to integrate to arrive in the general relativity regime increases rapidly with ϕ_B . Fortunately, as we will see below, this is the most interesting portion of the allowed values of ϕ_B . These simulations show that Ψ_o remains sharply peaked on an effective trajectory [7]. Since there is no obvious reason why this should not continue for higher ϕ_B values, it is instructive to examine all effective trajectories without restricting ourselves to kinetic energy dominated bounces. The trajectory would be compatible with the 7 year WMAP data *only if at the point at which H takes the value 7.83×10^{-6} , within the margin given by observational errors, $\varepsilon = 8 \times 10^{-3}$, and $\phi = 3.15$* . A surprising result is that this is in fact the case under a *very mild condition*: $\phi_B \geq 0.93$ [9]. Note that this result is much stronger than the qualitative ‘attractor behavior’ of inflationary trajectories because it is quantitative and tuned to the details of the WMAP observations. (For details, see [9]).

To make contact with the WMAP observations, we need to find k_* and the time $\tilde{\eta}(k_*)$ at which the mode with co-moving wave number k_* exits the Hubble horizon during inflation. For this, it is simplest to fix the scale factor at the bounce and we will choose the convention $a_B = 1$. (Note that this is very different from $a_{\text{today}} = 1$ often used in cosmology.) Then, along each dynamical trajectory one locates the point at which the Hubble parameter takes the value $H = 7.83 \times 10^{-6}$ (and makes sure that at this time ε and ϕ are given by (2) and (3) within observational errors). One calls the conformal time at which this occurs $\tilde{\eta}(k_*)$ and numerically calculates the scale factor $a(\tilde{\eta}(k_*))$ at this time. Then, the value of the co-moving momentum k_* of this mode is determined by the fact that this mode exits the Hubble radius at time $\tilde{\eta}(k_*)$. Thus, one asks that the *physical* wave number of this mode should equal the Hubble parameter: $k/a(\tilde{\eta}(k_*)) = H(\tilde{\eta}(k_*))$. Table 1 shows the values of k_* , the physical wave length of the mode at the bounce time, the *proper* time $\tilde{t}(k_*)$ at which the mode exits the Hubble horizon, and the number of e-foldings between the bounce and time $\tilde{t}(k_*)$ for a range of values of ϕ_B which turns out to be physically most interesting. (For details, see [13]).

- *Evolution of Perturbations:* Preliminary numerical simulations were first carried out using four different states ψ at the bounce, satisfying the initial conditions discussed in section 3. They showed that the results are essentially insensitive to the choice. Then detailed and much higher precision simulations were carried out using $|\psi\rangle = |0_{\text{obv}}\rangle$, the ‘obvious vacuum of 4th adiabatic order’, at the bounce because, as mentioned before, this state has a number of attractive properties. These simulations revealed an unforeseen behavior: the power spectra for scalar and tensor perturbations are largely insensitive to the value of ϕ_B . However, recall that there is finite window $(k_o, 2000k_o)$ of co-moving modes that can be seen in the CMB. Because of the pre-inflationary dynamics, the value of k_* —and hence of k_o — does depend on ϕ_B and rapidly increases with ϕ_B . (See Table 1.) Therefore, the window of ob-

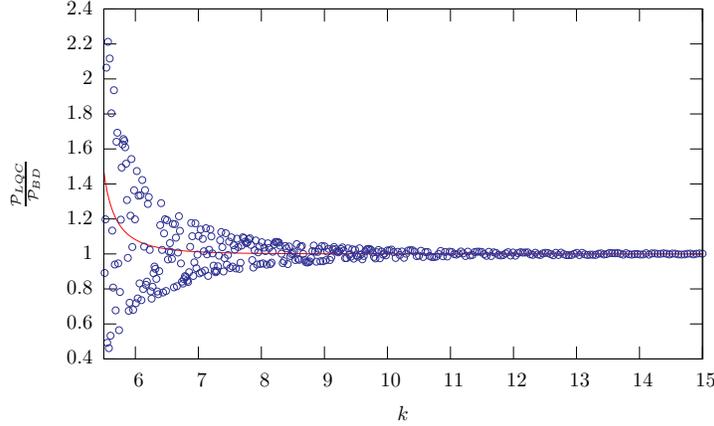


Fig. 3 Ratio of the LQC power spectrum for curvature perturbations in the scalar modes to that predicted by standard inflation (source [13]). For small k , the ratio oscillates very rapidly. The (red) solid curve shows averages over (co-moving) bins with width $0.5 \ell_{\text{pl}}^{-1}$.

servable modes is sensitive to the value of ϕ_{B} and moves steadily to the right as ϕ_{B} increases.

Fig. 3 shows the plot of the ratio $\mathcal{P}_{\mathcal{R}}^{\text{LQC}} / \mathcal{P}_{\mathcal{R}}^{\text{BD}}$ of the LQC power spectrum to the standard inflationary one for curvature perturbations \mathcal{R} of the scalar modes. The (blue) circles are the data points. The LQC power spectrum has very rapid oscillations (whose amplitudes decay quickly with k) which descend to the ratio that is plotted. Since observations have only a finite resolution, to compare with data it is simplest to average over small bins. We used bins which, at the bounce, corresponds to a band-width in physical wave numbers of $0.5 t_{\text{pl}}^{-1}$. The result is the solid (red) line. We see that the two power spectra agree for $k \gtrsim 6.5$ but LQC predicts an enhancement for $k \lesssim 6.5$. We will now comment on these features.

Let us first note that the LQC power spectrum in this plot uses the value $\phi_{\text{B}} = 1.15$. As Table 1 shows, the corresponding k_{\star} is 9.17. At this value, the two power spectra are identical, whence the amplitude and the spectral index obtained from the LQC evolution at $k = k_{\star}$ agrees with the values (1) observed by WMAP. However, as we remarked, for $k \lesssim 6.5$, the LQC prediction departs from that of standard inflation. These low k values correspond to $\ell \lesssim 22$ in the angular decomposition used by WMAP for which the error bars are quite large. Therefore, although the LQC power spectrum differs from the standard one in this range, both are admissible as far as the current observations are concerned.

What is the physics behind the enhancement of the LQC power spectrum for $k \lesssim 6.5$? And where does this specific scale come from? This enhancement is due to pre-inflationary dynamics. At the bounce, the scalar curvature has a universal value in LQC which sets a scale $k_{\text{LQC}} \approx 3.21$. Modes with $k \gg k_{\text{LQC}}$ experience negligible curvature during their pre-inflationary evolution while those with k comparable to k_{LQC} or less do experience curvature and therefore get excited. These are general

physical arguments and one needs numerical simulations to determine exactly what ‘much greater than’ and ‘comparable to’ means. The simulations show that modes with $k \gtrsim 2k_{\text{LQC}}$ already satisfy the ‘much greater than’ criteria. They are not excited and for them the LQC state ψ at the onset of inflation is virtually indistinguishable from the BD vacuum. That is why the two power spectra are essentially the same for $k \gtrsim 2k_{\text{LQC}}$. But for modes with $k \lesssim 2k_{\text{LQC}}$ the LQC state ψ has excitations over the BD vacuum whence there is an enhancement of the power spectrum.

What happens if we change ϕ_B ? As we remarked above, the prediction of the LQC power spectrum is pretty insensitive to the value of ϕ_B but the window in the k space spanned by modes which are observable in the CMB changes, moving to the right as ϕ_B increases. Now, as Table 1 shows, if $\phi_B > 1.2$, we have $k_o > 6.5$, whence none of the observable modes would be excited during the pre-inflationary evolution. In this case, at the onset of the slow roll, the LQC state ψ would be indistinguishable from the BD vacuum, whence all LQC predictions would agree with those of standard inflation. Thus, there is a narrow window, $0.93 \leq \phi_B \leq 1.2$ for which the background Ψ_o admits the desired slow roll phase and yet LQC predictions for future observations can differ from the standard ones. One example is given by a consistency relation $r = -8n_t$ in standard inflation, where $r = 2\mathcal{P}_{\mathcal{T}}^{\text{BD}} / \mathcal{P}_{\mathcal{R}}$ is the tensor to scalar ratio and n_t is the spectral index for tensor modes. This relation is significant because it does not depend on the form of inflationary potential. It turns out that r does not change in LQC but n_t does, whence this standard consistency relation is modified. Future observations would be able to test for such departures. There is also a systematic study of the effect that excitations over the BD vacuum can have on non-Gaussianities [26, 27, 28, 29]. Furthermore, it has been recently pointed out that these non-Gaussianities could be seen in the galaxy correlation functions and also in certain distortions in the CMB [61, 62, 63]. Thus, there are concrete directions in which cosmological observations could soon start probing effects that originate at the Planck scale. (For further details, see [13]).

- *Self Consistency*: Finally, let us discuss the issue of self-consistency of the truncation scheme, i.e., the issue of whether the test field approximation continues to hold under evolution. This issue is quite intricate and had remained unexplored because of two different issues. The first issue is conceptual: it was not clear how to compute the renormalized energy density for the quantum fields $\hat{\mathcal{Q}}, \hat{\mathcal{F}}$ in a manner that is meaningful in the Planck regime. As discussed in section 5, we were able to construct this framework by ‘lifting’ the adiabatic renormalization theory on classical cosmological space-times to that on quantum geometries Ψ_o . The second set of difficulties comes from numerics: one requires very high accuracy and numerical precision. This is because i) the rapid oscillations of integrand of $\langle \psi | \hat{\rho} | \psi \rangle_{\text{ren}}$ in the k space make it difficult to evaluate the exact value of the renormalized energy density; and, ii) the background energy density itself decreases from Planck scale to 10^{-11} times that scale. Indeed, so far we have only managed to find an upper bound on the energy density in the perturbations, shown in Fig. 4. But this suffices to show that, for $\phi_B > 1.22$, our initial conditions at the bounce do give rise to a self-consistent solution $\Psi_o \otimes \psi$ throughout the evolution from the big bounce to the onset of slow roll. These solutions provide a viable extension of the standard infla-

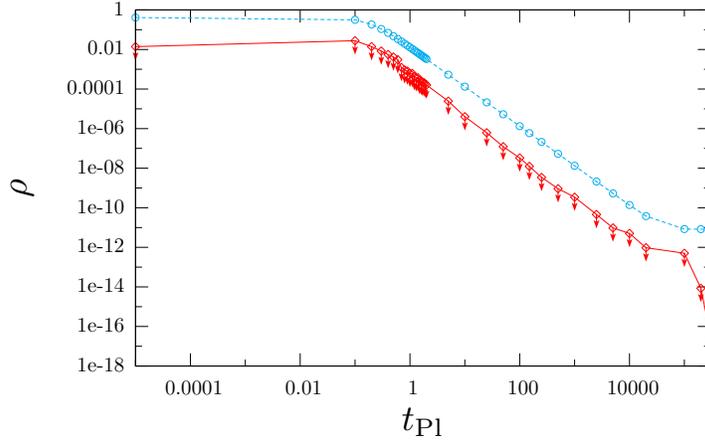


Fig. 4 For $\phi_B = 1.23m_{Pl}$, energy density in the background (upper curve) and an *upper bound* on the energy density in perturbations (lower curve) are plotted against time from the bounce to the onset of slow roll using Planck units (source [11]). The test field approximation holds across a change of 11 orders of magnitude in both quantities.

tionary scenario all the way to the Planck scale. The issue of whether one can push the value of ϕ_B to include the interesting domain $\phi_B < 1.2$ is still under investigation. (There are several aspects to this problem, including a better handling of the infrared regime, briefly discussed in [13].)

7 Summary and Discussion

I began in section 1 by making some suggestions: i) Progress in quantum gravity should be gauged by the degree to which an approach succeeds in overcoming limitations of general relativity; ii) The development of quantum theory, rather than general relativity, offers a better example to emulate in this endeavor; and, iii) As in quantum theory, it may be more fruitful to resolve concrete physical problems at the interface of gravity and quantum theory rather than focusing all efforts on obtaining a complete quantum gravity theory in one stroke. In sections 2 and 3 we saw that the very early universe offers an obvious arena for this task for both conceptual and practical reasons. Conceptually, the big bang is a prediction of general relativity in a regime in which the theory is not applicable, whence it is important to find out what really happened in the Planck regime. In practical terms, currently the early universe offers the best hope to confront quantum gravity theories with observations. In particular, we saw that the inflationary paradigm has been highly successful in accounting for the inhomogeneities in the CMB—and hence accounting for the large scale structure of the universe—but it has several limitations. In sections 4 - 6, I summarized how the limitations related to the Planck scale physics

are being addressed in LQG. Specifically, by using the truncation strategy of LQG, over the last six years it has been possible to extend the inflationary paradigm all the way to the deep Planck regime. (For other treatments of pre-inflationary dynamics within LQG, see e.g. [64, 65].)

The first finding is that the big bang singularity is resolved in LQC and replaced by the big bounce. Since quantum physics—including quantum geometry—is regular at the big bounce, it is natural to specify initial conditions for the quantum state Ψ_o that encodes the background, homogeneous quantum geometry, as well as for ψ that describes the quantum state of perturbations. Physically, the initial conditions amount to assuming that the state $\Psi_o \otimes \psi$ at the bounce should satisfy ‘quantum homogeneity’. More precisely, at the bounce one focuses just on that region which expands to become the observable universe and demands that it be homogeneous except for the inevitable quantum fluctuations that one cannot get rid of even in principle. Now, because of the pre-inflationary and inflationary expansion, the region of interest has a radius smaller than $\sim 10\ell_{\text{Pl}}$ at the bounce. But as has been emphasized in the relativity literature, this creates a huge fine tuning problem. For, to account for the impressive fact that inhomogeneities in the CMB are really tiny—just one part in 10^5 —the required homogeneity at the bounce has to be truly extraordinary. The standard inflationary paradigm is not really applicable at the Planck scale and, even if one were to ignore this fact, it does not have a natural mechanism to achieve this degree of homogeneity. In LQC, on the other hand, the big bang singularity is resolved precisely because there is an in-built repulsive force with its origin in the specific quantum geometry that underlies LQG. While this force is negligible when curvature is less than, say, 10^{-6} in Planck units, it rises spectacularly in the Planck regime, overcomes the huge classical gravitational attraction and prevents the big bang singularity. In more general models referred to in section 4, one finds a pattern: every time a curvature scalar enters the Planck regime, this repulsive force becomes dominant and dilutes that curvature scalar, preventing a singularity (see e.g. [15]). This opens the possibility that the ‘dilution effect’ of the repulsive force may be sufficient to create the required degree of homogeneity on the scale of about $10\ell_{\text{Pl}}$, thereby accounting for the assumed ‘quantum homogeneity’. If this idea could be developed in detail, dynamics of the pre-bounce universe will leave no observable effects, providing a clear-cut case for specifying initial conditions at the bounce. Of course, the pre-bounce dynamics will still lead to inhomogeneities at larger scales on the bounce surface but they would have Fourier modes whose physical wave length is much larger than the radius of the observable universe. Therefore, they would not be in the observable range; in the truncated theory considered here, they would be absorbed in the quantum geometry of the homogeneous background. This ‘dilution mechanism’ and other issues related to initial conditions are likely to be a center of activity in the coming years.

As we saw in sections 5 and 6, we now have a conceptual framework and numerical tools to evolve these initial conditions all the way from the bounce to the onset of slow roll. The result depends on where one is in the parameter space that is labeled by the value ϕ_{B} of the inflaton at the bounce. For a very large portion of the parameter space we obtain the following three features: i) Some time in its

future evolution, the background geometry encounters a slow roll phase that is compatible with the 7 year WMAP observations; ii) At the onset of this slow roll, the state ψ of perturbations is essentially indistinguishable from the BD vacuum used in standard inflation; and iii) the back reaction due to perturbations remains negligible throughout pre-inflationary dynamics in which the background curvature falls by some 11 orders of magnitude, justifying the underlying ‘truncation approximation’. Thus, for this portion of the parameter space, we have a self-consistent extension of the standard inflationary paradigm.

There is, however, a small window in the parameter space for which the feature i) is realized but the initial state at the onset of inflation contains an appreciable number of BD excitations. This number is within the current observational limits. But the presence of these excitations signals new effects such as a departure from the inflationary ‘consistency relation’ involving both scalar and tensor modes and a new source of non-Gaussianities. These could be seen in future observational missions [61, 62, 63]. The physical origin of these effects can be traced back to a new energy scale k_{LQC} defined by the universal value of the scalar curvature at the bounce. Excitations with $k \lesssim 2k_{\text{LQC}}$ are created in the Planck regime near the bounce. It turns out that if the number N of e-foldings in the scale factor a between the bounce and $\tilde{\eta} = \tilde{\eta}_{k^*}$ is less than 15, then the modes which are excited would be seen in the CMB. This occurs only in the small window of parameter space referred to above. Since the window is very small, the ‘a priori probability’ that one of these values of ϕ_{B} is realized in Nature would seem to be tiny. However, one can turn this argument around. Should these effects be seen, the parameter space would be narrowed down so much that very detailed calculations would become feasible. In either case, it is rather exciting that the analysis relates initial conditions and Planck scale dynamics with observations, thereby expanding the reach of cosmology to the earliest moment in the deep Planck regime.

Even when a self-consistent solution $\Psi_o \otimes \psi$ to the truncated theory exists, how would it fit in full LQG? Recall the situation in classical general relativity. In cosmology as well as black hole physics, one routinely expects first order perturbations whose back reaction is negligible to provide excellent approximations to the phenomenological predictions of the exact theory. I see no obvious reason why the situation would be different in quantum gravity. As a simple example to illustrate the general viewpoint, consider the Dirac solution of the hydrogen atom. Since one assumes spherical symmetry prior to quantization, this truncation excludes photons from the beginning. Therefore, at a conceptual level, the Dirac description is *very* incomplete. Yet, as far as experiments are concerned, it provides excellent approximations to answers provided by full QED until one achieves the accuracy needed to detect the Lamb shift. I expect the situation to be similar for our truncated theory: Conceptually it is surely quite incomplete vis a vis full LQG, but the full theory will provide only small corrections to the observable effects.

To conclude, let me emphasize that there was no a priori reason to anticipate either of the two main conclusions —the extension of standard inflation to the Planck regime for much of the parameter space and deviations from some of its predictions in a narrow window. Indeed, it would *not* have been surprising if the pre-inflationary

dynamics of LQC was such that the predicted power spectra were observationally ruled out for the ‘natural’ initial conditions we used at the bounce, or, if the self-consistency of truncation had failed quite generally because of the Planck scale dynamics. Indeed, this could well occur in generic bouncing scenarios, e.g. in situations in which the expansion between the bounce and the surface of large scattering is not sufficiently large for the modes observed in the CMB to have wave lengths smaller than the curvature radius throughout this evolution.

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